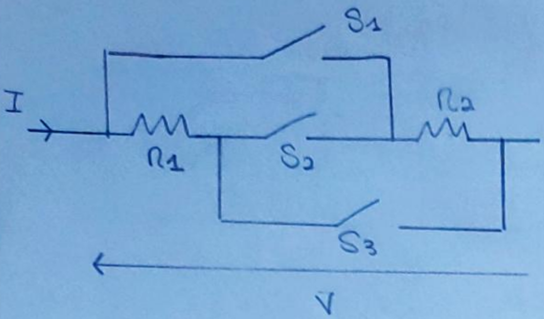
# TUTORIAL 2




- $S_1, S_2, S_3 \Rightarrow$ aperti
  $I = 0$
  $R_{AB} = \infty$

- $S_1, S_2 \Rightarrow$ aperti
  $S_3 \Rightarrow$ chiuso
  $R_{AB} = R_1$

- $S_1, S_3 \Rightarrow$ aperti
  $S_2 \Rightarrow$ chiuso
  $R_{AB} = R_1 + R_2$

- $S_1 \Rightarrow$ aperto
  $S_2, S_3 \Rightarrow$ chiuso
  $R_{AB} = R_1$ il lato con $R_2$ e' cortocircuitato, la $I$ passa tutta nel cortocircuito

- $S_1 \Rightarrow$ chiuso
  $S_2, S_3 \Rightarrow$ aperto
  $R_{AB} = R_2$

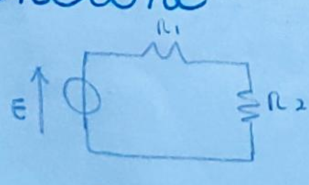
- $S_1, S_3 \Rightarrow$ chiusi
  $S_2 \Rightarrow$ aperto
  $R_{AB} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$

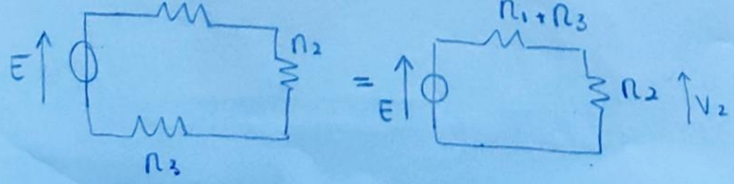
- $S_1, S_2 \Rightarrow$ chiusi
  $S_3 \Rightarrow$ aperto
  $R_{AB} = R_2$

- $S_1, S_2, S_3 \Rightarrow$ chiusi
  $R_{AB} = 0$

## partitore di tensione

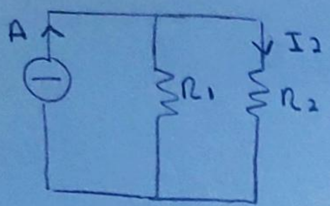
$$V_2 = E \cdot \frac{R_2}{R_1 + R_2}$$



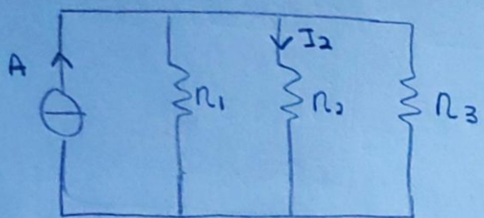




$$V_2 = E \frac{R_2}{R_1 + R_2 + R_3}$$

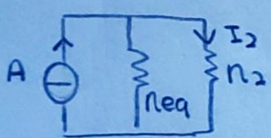
# derivatore di corrente:




$$I_2 = A \cdot \frac{R_1}{R_1 + R_2}$$




riconosco il collegamento in || di $R_1$ e $R_3$

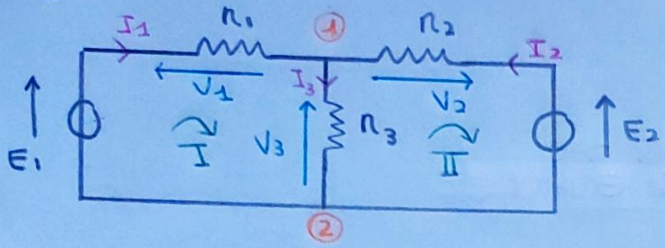



$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_3}\right)^{-1}$$

$$I_2 = A \cdot \frac{R_{eq}}{R_{eq} + R_2}$$

# metodo generale

1




$R_1 = R_2 = 20\,\Omega$
$R_3 = 80\,\Omega$
$E_1 = 30V$
$E_2 = 60V$

$I_1 + I_2 - I_3 = 0$
$E_1 - V_1 - V_3 = 0$
$V_3 + V_2 - E_2 = 0$

$V_1 = R_1 I_1$
$V_2 = R_2 I_2$
$V_3 = R_3 I_3$

$E_1 - R_1 I_1 - R_3 I_3 = 0$
$R_3 I_3 + R_2 I_2 - E_2 = 0$

$$\begin{bmatrix} 1 & 1 & -1 \\ R_1 & 0 & R_3 \\ 0 & R_2 & R_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ E_1 \\ E_2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 20 & 0 & 80 \\ 0 & 20 & 80 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 30 \\ 60 \end{bmatrix}$$

$$I_1 = -\frac{1}{3600} \begin{vmatrix} 0 & 1 & -1 \\ 20 & 0 & 80 \\ 0 & 20 & 80 \end{vmatrix}$$ $I_1 = -0{,}5A$

$$I_2 = -\frac{1}{3600} \begin{vmatrix} 1 & 0 & -1 \\ 20 & 30 & 80 \\ 0 & 60 & 80 \end{vmatrix}$$ $I_2 = 1A$

$$I_3 = -\frac{1}{3600} \begin{vmatrix} 0 & 1 & 0 \\ 20 & 0 & 30 \\ 0 & 20 & 60 \end{vmatrix}$$ $I_2 = 0{,}5A$

$V_1 = -10V$
$V_2 = 20V$
$V_3 = 40V$

$P_{E1} = E_1 \cdot I_1$

$P_{E1} = 30 \cdot (-0,5) = -15W$ ⟶ potenza negativa del generatore ⇒ UTILIZZATORE

$P_{E2} = E_2 \cdot I_2$

$P_{E2} = 60\ W$ → potenza positiva del generatore ⇒ GENERATORE

$P_1 = V_1 \cdot I_1 = R_1 \cdot I_1^2$

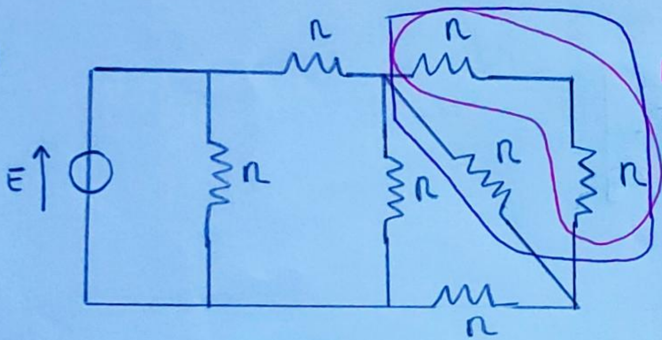
$P_1 = 5W$

$P_2 = 20W$

$P_3 = 20W$

$\Sigma P = 0$

② 

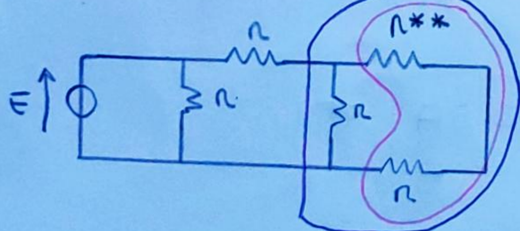



$E = 10V$

$R = 10\ \Omega$

$R^* = 2R$

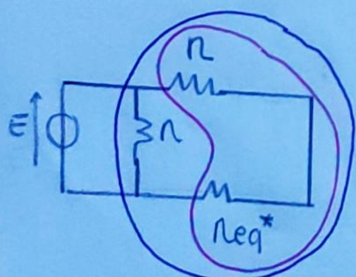
$R^{**} = \left(\frac{1}{R} + \frac{1}{2R}\right)^{-1} = \frac{2}{3}R$




$R_{eq} = \frac{2}{3}R + R = \frac{5}{3}R$

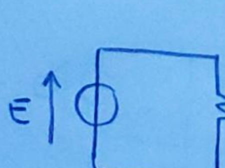
$R_{eq}^* = \left(\frac{1}{R} + \frac{1}{R_{eq}}\right)^{-1}$

$R_{eq}^* = \left(\frac{1}{R} + \frac{5}{3R}\right)^{-1} = \frac{5}{8}R$




$R = \frac{5}{8}R + R = \frac{13}{8}R$

$R'_{eq} = \left(\frac{8}{13R} + \frac{1}{R}\right)^{-1} = \frac{13}{21}R$



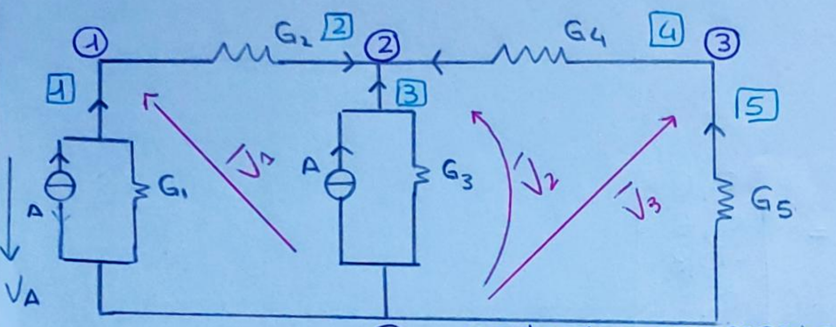

$R'_{eq} = \frac{13}{21}R$

$I = \frac{E}{R_{eq}}$

$I = \frac{10}{13 \cdot 10} \cdot 21 = 1,615\ A$

# TUTORIAL 3

## potenziali di nodo




④ -> nodo di riferimento.

$G_1 = 2\,S$
$G_2 = 1\,S$
$G_3 = 3\,S$
$G_4 = 1\,S$
$G_5 = 1\,S$

$A_1 = 1A$
$A_2 = 10\,V_1$

$$C_t = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$$G = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 10 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\bar{G} = C \cdot G \cdot C^T$$

$$\bar{G} = \begin{bmatrix} 3 & -1 & 0 \\ 9 & 5 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

$G_m - 1$

[per trovare l'intervallo dei valori Gm accettati calcolo il det e lo pongo ≠ 0, lo faccio lasciando l'incognita Gm.]

$$\bar{I} = CGE - CA$$

$$E = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad A = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Il lato 3 non ha corrente impressa perche' e' comandato.

$$\bar{I} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\bar{V} = \bar{G}^{-1} \cdot \bar{I}$$

$$\bar{V} = \begin{bmatrix} 0{,}2 \\ -0{,}4 \\ -0{,}2 \end{bmatrix}$$

$V = C^T \cdot \bar{V}$

$$V = \begin{bmatrix} -0{,}2 \\ 0{,}5 \\ 0{,}4 \\ 0{,}2 \\ 0{,}2 \end{bmatrix}$$

$I = A + GV$

$I = A + G(V - E)$

$$I = \begin{bmatrix} 0{,}6 \\ 0{,}6 \\ -0{,}8 \\ 0{,}2 \\ 0{,}2 \end{bmatrix}$$

$P_{A1} = V_1 \cdot I_1$

$P_{A1} = -0{,}12\ W$ generatore

$\Sigma P = 0$

$P_2 = 0{,}36\ W$

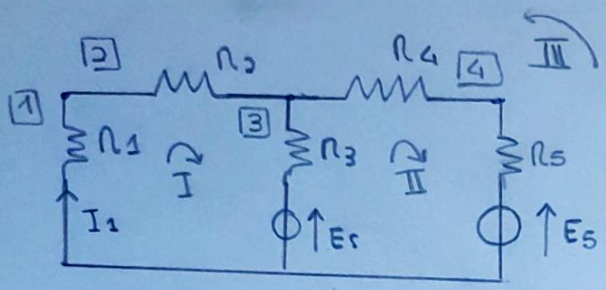
$P_4 = 0{,}04\ W$

$P_5 = 0{,}04\ W$

$P_{A2} = -0{,}32\ W$ generatore

# TUTORIAL 4

metodo delle correnti di maglia:




$R_1 = R_2 = R_3 = R_4 = R_5 = 1\,\Omega$

$E_3 = 2I_1$

$E_5 = 1V$

$$M_t = \begin{bmatrix} 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 \\ \cancel{-1} & \cancel{-1} & \cancel{0} & \cancel{1} & \cancel{1} \end{bmatrix}$$

-> scarico la maglia esterna e ottengo

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 \end{bmatrix} = M_r$$

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$\bar{R} = M_r \cdot R \cdot M_r^T$

$$\bar{R} = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}$$

$$E = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

$\bar{V} = M \cdot R \cdot A - ME$

$\bar{I} = \bar{R}^{-1} \cdot \bar{V}$

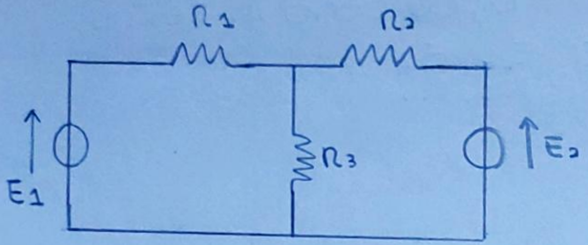
$$\boxed{\begin{aligned} I &= M_r^T \cdot \bar{I} \\ V &= E + R(I - A) \end{aligned}}$$

$\Sigma VI = 0$ => Teorema della conservazione delle potenze.

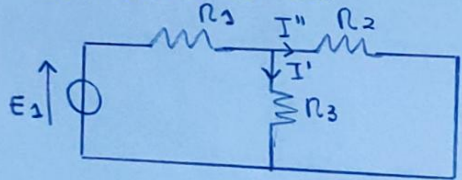
La sommatoria delle potenze deve essere nulla.

# TUTORIAL 5

## sovrapposizione effetti → si può applicare su circuiti lineari




• effetto di E1




$R_2 // R_3$

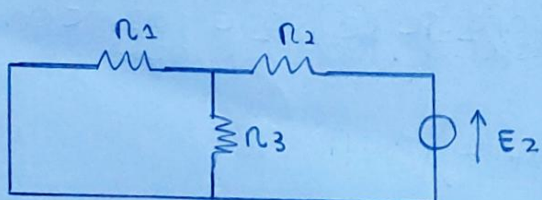
→ spengo E e ottengo un lato di corto circuito

$$R_{eq}^{E_1} = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

$$I_{E_1} = \frac{E}{R_{eq_{E_1}}}$$

$$I_1' = I_{E_1} \cdot \frac{R_2}{R_3 + R_2} \qquad I_1'' = I_E - I'$$

• effetto E2




$$R_{eq}^{E_2} = R_2 + \left(\frac{1}{R_1} + \frac{1}{R_3}\right)^{-1}$$

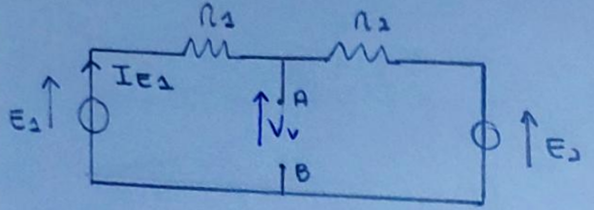
$$I_{E_2} = \frac{E}{R_{eq_{E_2}}}$$

$$I_2' = I_{E_2} \cdot \frac{R_1}{R_3 + R_1} \qquad I_2'' = I_{E_2} - I_2'$$

$$I_3 = I_1' + I_2'$$

# teorema Thevenin:

calcolare la $V_v$ ai morsetti che si stanno considerando




Escludo $R_3$.
Tutti i bipoli sono in serie
Noti $E_1, E_2$
$R_1, R_2$

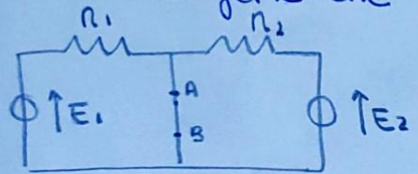
$$E_1 - I_{E1}(R_1 + R_2) - E_2 = 0$$

$$I_{E1} = \frac{E_1 - E_2}{R_1 + R_2}$$

$$E_1 - R_1 I_{E1} = V_v$$

come calcolare $R_{AB}$:

1) metodo generale => cortocircuito AB




$E_1 - R_1 I_{E1} = 0$ $\quad I_{E1} = \frac{E_1}{R_1}$

$E_2 - R_2 I_{E2} = 0$ $\quad I_{E2} = \frac{E_2}{R_2}$

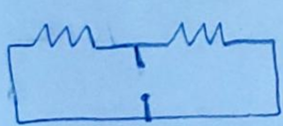
$I_{cc} = I_{E1} + I_{E2}$

$R_{AB} = \frac{V_v}{I_{cc}}$ → precedentemente calcolata

2) metodo particolare

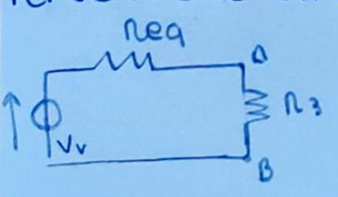
Si escludono tutti i generatori e si guardano come sono connesse le varie resistenze rispetto i morsetti AB.

E → cortocircuito
A → aperto



$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

Riscrivo il circuito

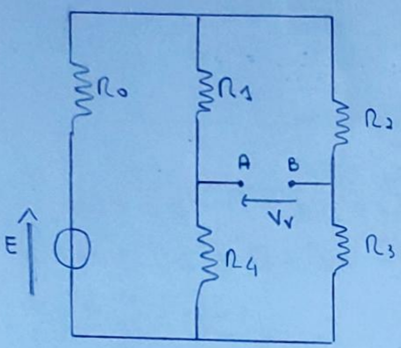



$$V_v - R_{AB} I_3 - R_3 I_3 = 0$$

$$I_3 = \frac{V_v}{R_{AB} + R_3}$$

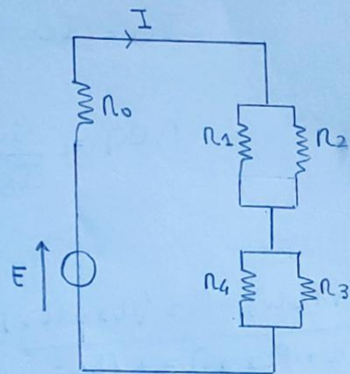
# TUTORIAL 6

## Bipolo di Norton




$I_{cc} = I_1 - I_4$

① $I_{cc}$:




$$I_1 = I \cdot \frac{R_2}{R_1+R_2}$$

$$I_4 = I \frac{R_3}{R_3+R_4}$$

$$I_{cc} = I\left(\frac{R_2}{R_1+R_2} + \frac{R_3}{R_3+R_4}\right)$$

$$I = \frac{E}{R_{eq}^E}$$

$$R_{eq}^E = R_0 + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} + \left(\frac{1}{R_3} + \frac{1}{R_4}\right)^{-1}$$

$$R_{eq}^E = R_0 + \frac{R_1R_2}{R_1+R_2} + \frac{R_3R_4}{R_3+R_4}$$

$$I_{cc} = \frac{E}{R_{eq}^E}\left(\frac{R_2R_3 + R_2R_4 - R_1R_3 - R_2R_3}{(R_1+R_2)(R_3+R_4)}\right)$$

② $V_V$; $R_{TH}$

$$R_{AB} = \frac{V_V}{I_{cc}}$$

TROVO LA NUOVA RESISTENZA EQUIVALENTE RISPETTO I MORSETTI AB => quando ai morsetti AB c'è un aperto.

$$I_E = \frac{E}{R_{eq}'}$$

$$R_{eq}' = R_0 + \left(\frac{1}{R_1+R_4} + \frac{1}{R_3+R_2}\right)^{-1}$$

$$I_1 = I_E \frac{R_2+R_3}{R_2+R_3+R_4+R_1}$$

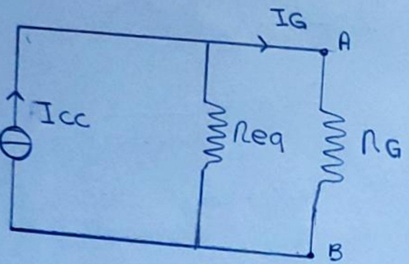
$$I_2 = I_E \frac{R_1+R_4}{R_1+R_2+R_3+R_4}$$

KVL: $R_1 I_1 - R_2 I_2 + V_V = 0$

$$V_V = R_2 I_2 - R_1 I_1$$

$$V_V = I_E \, \frac{R_2(R_1+R_4) - R_1(R_2+R_3)}{R_1+R_2+R_3+R_4}$$

Abbiamo identificato il bipolo di Norton:




$$I_G = I_{cc} \cdot \frac{R_{eq}}{R_{eq}+R_G}$$

$$R_{eq} = \frac{V_V}{I_{cc}}$$

$$V_V = \frac{E}{R'_{eqE}} \cdot \frac{R_2(R_1+R_4) - R_1(R_2+R_3)}{R_1+R_2+R_3+R_4}$$

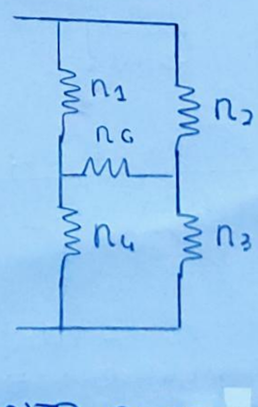
$R_{eqE} \neq R'_{eqE}$

$$I_{cc} = \frac{E}{R_{eqE}} \, \frac{R_2R_4 - R_1R_3}{(R_1+R_2)(R_3+R_4)}$$

$$V_G = R_G \cdot I_G$$

$I_G = 0 \Rightarrow R_1R_3 = R_2R_4$

Regolo $R_2$ in modo tale che $I_G$ si annulli

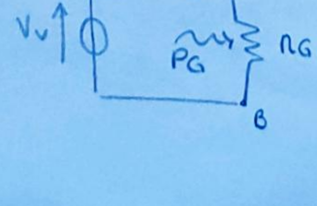



$R_2$ regolabile.

PRINCIPIO DI MASSIMO TRASFERIMENTO DI POTENZA:

$R_G = R_{eq}$

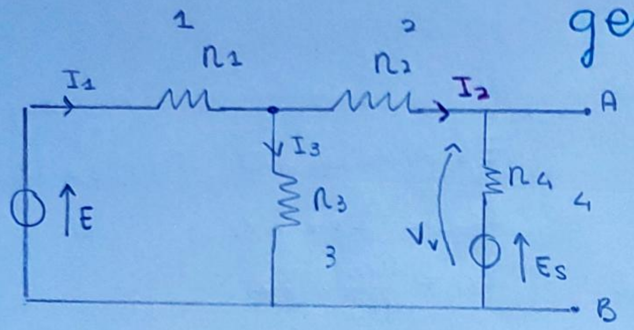
$$P_G\Big|_{R_G=R_{eq}} = \frac{V_V^2}{4R_{eq}}$$




$$\frac{\partial P_G}{\partial R_G}\Big|_{R_G=R_{eq}} = 0$$

# Bipolo equivalente

## generatore comandato




$E_s = R_m I_2$

Nel caso in cui $I_A = 0$ quindi ho un aperto il lato 2 e il lato 4 sono in serie.

$I_4 = I_2$ perche' $R_2$ e' in serie con $R_4$.

$E - R_1 I_1 - R_3 I_3 = 0$

$E - R_1 I_1 - R_2 I_2 - R_4 I_2 - E_s = 0$

$I_1 = I_2 + I_3$

$I_2 = I_4$

$R_1 I_1 + R_3 I_3 = E$

$R_1 I_1 + R_2 I_2 + R_4 I_2 + R_m I_2 = E$

$I_1 - I_2 - I_3 = 0$

$$\begin{pmatrix} R_1 & 0 & R_3 \\ R_1 & R_2 + R_4 + R_m & 0 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} E \\ E \\ 0 \end{pmatrix}$$

$$I_2 = \frac{1}{\Delta} \begin{vmatrix} R_1 & E & R_3 \\ R_1 & E & 0 \\ 1 & 0 & -1 \end{vmatrix}$$

$\Delta \to$ riferico alla 1° matrice la seconda matrice la calcolo sostituendo l'ultima matrice nella colonna corrispondente al risultato che sto cercando.

$\Delta = -10^4$

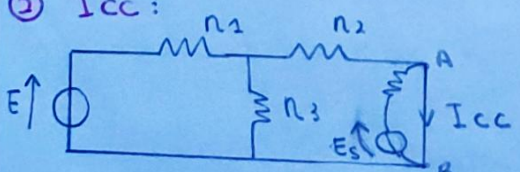
$I_2 = 10^4 (-R_3 E) = 2A$

① $V_v$:

$V_v = R_4 I_2 + E_s$

$V_v = I_2 (R_4 + R_m)$ $\qquad V_v = 100\,V$

② $I_{cc}$:



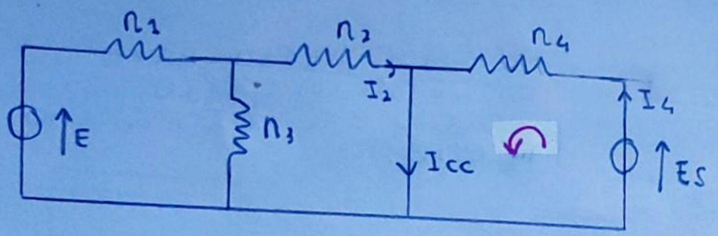

$R_{eq} = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$

$R_{eq} = R_1 \; \frac{R_2 R_3}{R_2 + R_3}$

$I_1 = I_E = \frac{E}{R_{eq}}$

$$I_2 = I_1 \cdot \frac{r_3}{r_2 + r_3}$$

$$I_2 = \frac{E}{r_{eq}} \cdot \frac{r_3}{r_2 + r_3}$$




$$E_s - r_4 I_4 = 0 \qquad r_m I_2 = r_4 I_4$$

$$I_{cc} = I_2 + I_4$$

$$I_4 = \frac{r_m}{r_4} \cdot I_2$$

$$I_{cc} = I_2 \left(1 + \frac{r_m}{r_4}\right)$$

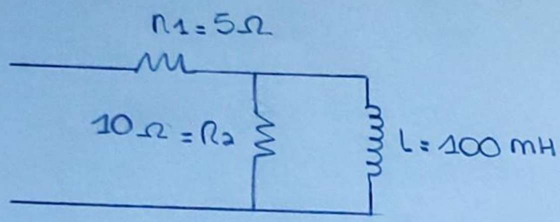
$$r_{AB} = \frac{V_v}{I_{cc}}$$

# TUTORIAL 7

## regine periodico alternato sinosoidale

GRANDEZZE ISOFREQUENZIALI ⇒ se hanno la stessa pulsazione $\omega$.

① 




$\omega = 100$ rad/s

$$Z = R_1 + \left(\frac{1}{R_2} + \frac{1}{j\omega L}\right)^{-1}$$

$$Z = R_1 + \frac{j\omega L R_2}{j\omega L + R_2}$$

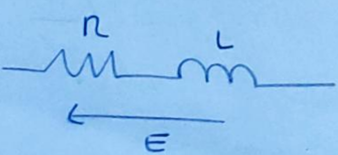
$$Z = 5 + \frac{100j}{10 + 10j}$$

$$\bar{Z} = 5 + \left[\frac{100j}{10+10j} \cdot \frac{10-10j}{10-10j}\right]$$

$$\bar{Z} = 5 + \frac{100j(10-10j)}{200} = 10 + 5j \ \Omega$$

$$\bar{Y} = \frac{1}{\bar{Z}} = \frac{1}{10+5j} = 80 - 40j \text{ mS}$$

② 




$R = 100\ \Omega$
$L = 100$ mH
$\omega = 100$ rad/s

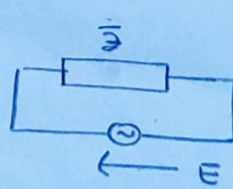
$e(t) = 10\sqrt{2}\cos(\omega t)$
$\bar{E} = 10 \angle 0°$ V

$$\bar{Z} = R + j\omega L = 100 + 10j \ \Omega$$

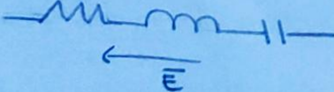
$$\bar{Y} = \frac{1}{\bar{Z}}$$

$$I = \frac{\bar{E}}{\bar{Z}}$$

$$I = \bar{E} \cdot \bar{Y}$$




③ 




$$\bar{Z} = R + j\omega L - \frac{j}{\omega C}$$

$$\bar{Y} = \frac{1}{\bar{Z}}$$

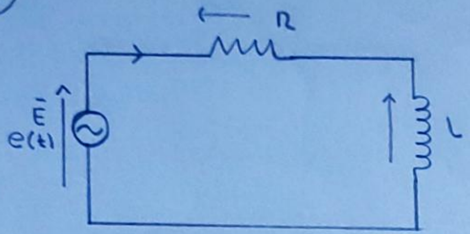
$e(t) = 10\sqrt{2}\cos(\omega t)$

$$I = \frac{\bar{E}}{\bar{Z}} = \bar{E} \cdot \bar{Y}$$

# TUTORIAL 8.

## regime p.a.s.

(1)




$R = 3\,\Omega$
$L = 40\,mH$
$e(t) = 10\sqrt{2}\cos(100t)$

$I = \dfrac{\bar{E}}{R + j\omega L}$

$\bar{E} - R\cdot\bar{I} - j\omega L\cdot\bar{I} = 0$

$\bar{I} = \dfrac{\bar{E}}{R + j\omega L}$

$V_R = R\cdot\bar{I}$

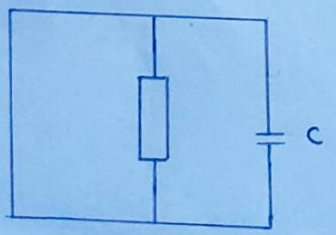
$V_L = j\omega L\cdot\bar{I}$

$P_R = V_R\cdot\bar{I}$ $\quad Q_R = 0 \quad \Rightarrow$ RESISTENZA

$P_L = 0$ $\quad Q_R = V_L\cdot\bar{I} \quad \Rightarrow$ INDUZIONE

Vale sempre il teorema che le somme delle potenze deve essere nullo.

(2)

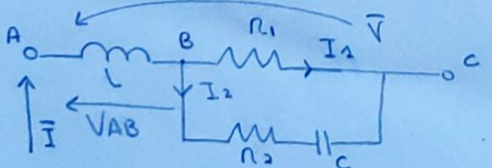



$Z_C = -\dfrac{j}{\omega C}$

$V_C = -\dfrac{j}{\omega C}\cdot I$

$P_C = V_C\cdot I$

(3)




$\bar{I} = \bar{I}_1 + \bar{I}_2$

$Z_{eq} = Z_L + \left(\dfrac{1}{R_1} + \dfrac{1}{Z_C + R_2}\right)^{-1}$

$Q_L = Z_L\cdot\bar{I}^2$
$P_1 = R_1\cdot\bar{I}_1^2$
$P_2 = R_2\cdot\bar{I}_2^2$
$Q_C = Z_C\cdot\bar{I}_2^2$

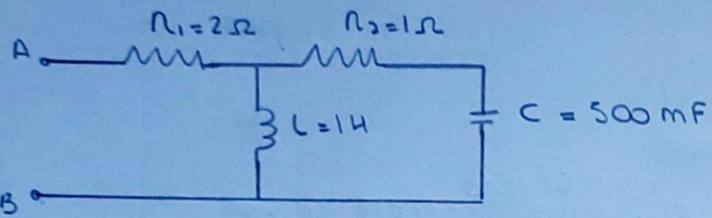
Nota $\bar{V} \Rightarrow \bar{I} = \dfrac{\bar{V}}{Z_{eq}}$

$V_{AB} = Z_L\cdot\bar{I}$

$V_{BC} = V - V_{AB}$

# TUTORIAL 9

①




$$Z_{AB} = R_1 + \left[ \frac{1}{jωL} + \frac{1}{R_2 - \frac{j}{ωC}} \right]^{-1}$$

$$Z_{AB} = \frac{3ω^4 - 6ω^2 + 8}{(2-ω^2)^2 + ω^2} + j\,\frac{ω(4-ω^2)}{(2-ω^2)^2 + ω^2}$$

$$ω_0 = \frac{1}{\sqrt{LC}}$$

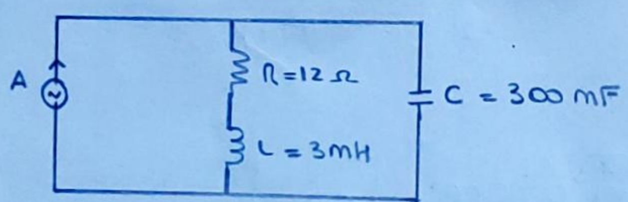
pulsazione di risonanza

↓

si riferisce al caso ideale, negli altri casi l'ordine di grandezza è uguale, varia di poco per la R

$\Rightarrow$ RISUONA PARALLELO

②




$$Y_{eq} = jωC + (R + jωL)^{-1}$$

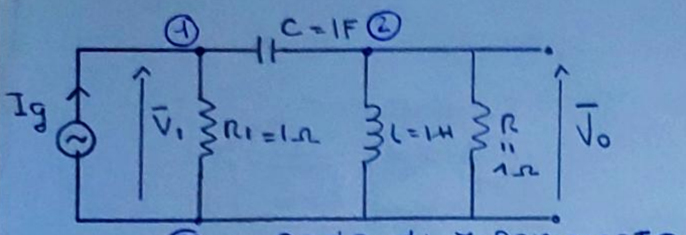
$\Rightarrow$ RISUONA PARALLELO

risonanza confrontabile con $1/\sqrt{LC}$

$ω_0 \Rightarrow$ pongo la parte immaginaria uguale 0

Risonanza minima $= 1/R$

③




③ → nodo di riferimento

ammettenza posto 11

matrice delle ammettenze

$$\begin{bmatrix} \frac{1}{R_1} + j\omega C & -j\omega C \\ -j\omega C & \frac{1}{R_2} + j\left(\omega C - \frac{1}{\omega L}\right) \end{bmatrix} \begin{bmatrix} \bar V_1 \\ \bar V_o \end{bmatrix} = \begin{bmatrix} \bar I_g \\ 0 \end{bmatrix}$$

ammettenze tra il nodo 1 e 2 con segno opposto

ammettenza 22

$$\bar V_o = \frac{1}{\Delta} \begin{vmatrix} 1 + j\omega & \bar I_g \\ -j\omega & 0 \end{vmatrix}$$

$$\bar V_o = \frac{-\omega^2 \bar I_g}{j\omega \cdot 2 - 2\omega^2 + 1}$$

sostituisco questa colonna nella seconda perchè $\begin{bmatrix} V_1 \\ V_o \end{bmatrix}$ → è nel secondo posto.

$$\frac{V_o}{I_g} = \frac{-\omega^2}{2\omega j - 2\omega^2 + 1}$$

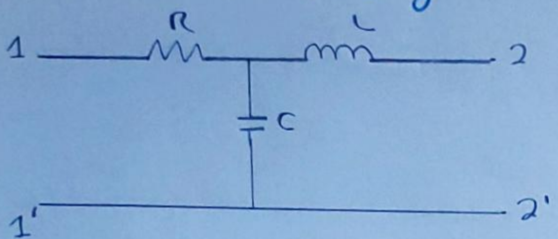
$$\downarrow$$

$$\frac{\omega^2}{\left[(1 - 2\omega^2)^2 + 4\omega^2 j\right]^{1/2}}$$

$1 - 2\omega^2 = 0 \qquad \omega = 1/\sqrt{2}$

# TUTORIAL 10

## parametri Z e Y




$R = 10\ \Omega$
$JWL = 20j\Omega$
$JWC = -30j$

1° metodo

$Z_{11} = R - \frac{J}{WC} \qquad Z_{11} = 10 - 30j\ \Omega$

$Z_{22} = JWL - J/WC \qquad Z_{22} = 20j - 30j = -10j\ \Omega$

$Z_{12} = Z_{21} = \left(\frac{V_2}{I_1}\right)_{I_2=0}$

$Z_{12} = Z_{21} = \frac{-J/WC \cdot \cancel{I_1}}{\cancel{I_1}} = -30j\ V$

$Z = \begin{bmatrix} 10-30j & -30j \\ -30j & -10j \end{bmatrix}$

$Y = \frac{1}{\Delta Z} \begin{bmatrix} -10j & 30j \\ 30j & 10-30j \end{bmatrix}$

$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$

$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$
$\parallel$
$Z_{12}$

$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$

matrice simmetrica

$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$

$Y = \frac{1}{\det Z} \begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{22} \end{bmatrix}$

# 2° metodo

$$\bar{Y}_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$$ → il lato 22' e' cortocircuitato

metto un generatore di corrente in 11'

$$\bar{Y}_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$ → il lato 11' e' cortocircuitato, metto un generatore di corrente in 11'

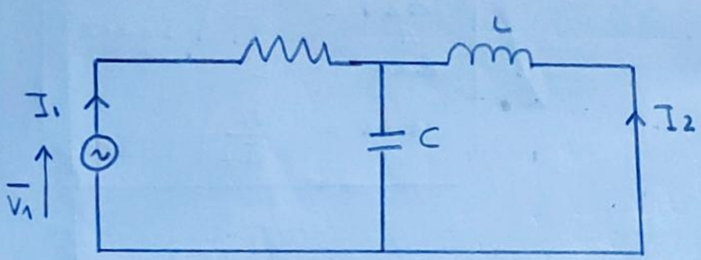
$$\bar{Y}_{21} = \bar{Y}_{12} = -\frac{I_1}{V_1}$$

## parametri h

$$\begin{cases} V_1 = h_{11} I_1 + h_{12} V_2 \\ I_2 = h_{21} I_1 + h_{22} V_2 \end{cases}$$

$$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0}$$

$$h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0}$$

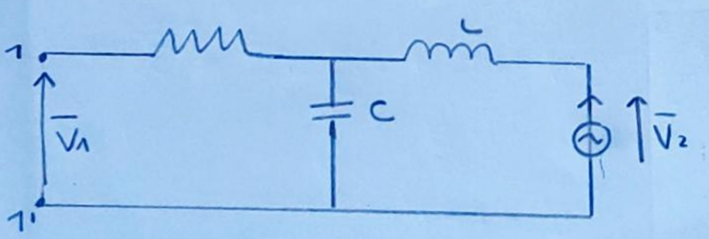



$$h_{11} = R + \left(\frac{1}{-J/\omega C} + \frac{1}{J\omega L}\right)^{-1}$$

$$h_{21} = -h_{12} \qquad h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0}$$

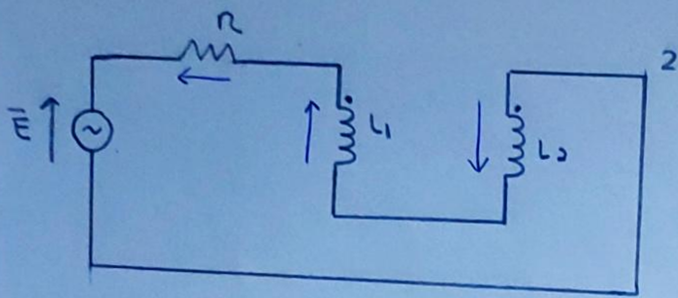
$$h_{22} = \left(\frac{1}{J\omega L} - \frac{1}{J/\omega C}\right)$$

$$h_{12} = \frac{-J/\omega C}{J\omega L - J/\omega C}$$




- $h_{21} = h_{12}$ ⇒ ADIMENSIONALE
- $h_{11}$ ⇒ $\Omega$
- $h_{22}$ ⇒ $\Omega^{-1}$

# mutua induttanza



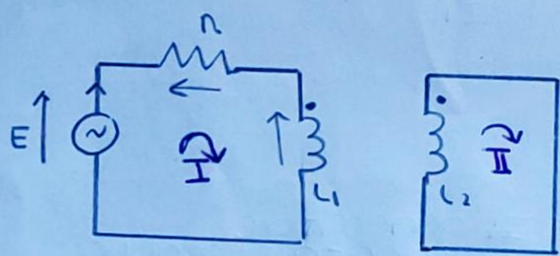

$Z_{eq}$ ⇒ non applicabile perché ho la mutua induttanza

non posso ricavarlo direttamente dal disegno

ricavo $Z_{eq}$ da questa equazione

$$\bar{E} = rI + \underbrace{j\omega L_1 I - j\omega M I}_{V_{L1}} + \underbrace{j\omega L_2 I - j\omega M I}_{V_{L2}}$$

$$\bar{E} = \underbrace{(r + j\omega L_1 + j\omega L_2 - 2j\omega M)}_{Z_{eq}} I$$




$$\underbrace{j\omega L_2 I_2}_{V_{L2}} + j\omega M I_1 = 0$$

$$I_2 = -\frac{M}{L_2} I_1$$

$$E = rI_1 + j\omega L_1 I_1 + j\omega M I_2$$

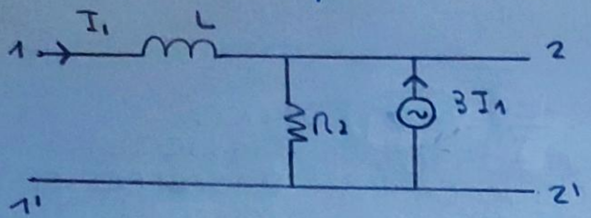
$$E = rI_1 + j\omega L_1 I_1 - j\omega \frac{M^2}{L_2} I_1$$

$$E = I_1 \underbrace{\left[r + j\omega \left(L_1 - \frac{M^2}{L_2}\right)\right]}_{Z_{eq}}$$

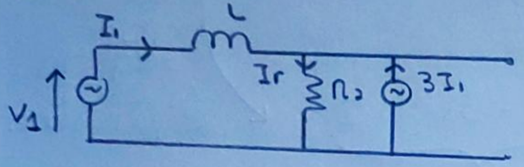
$$I_1 = \frac{E}{r + j\omega\left(L_1 - \frac{M^2}{L_2}\right)}$$

⇒ ricavo $I_1$ e $I_2 = -\frac{M}{L_2} I_1$

# doppio bipolo non reciproco




① porta 2 a vuoto




$I_r = I_1 + 3I_1 \qquad I_r = 4I_1$

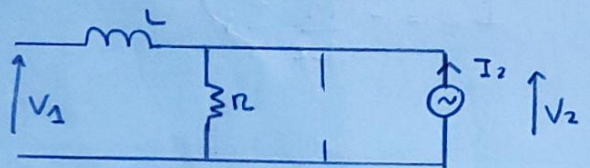
$V_1 = j\omega L\, I_1 + R \cdot 4I_1$

$V_2 = R\ 4I_1$

$Z_{11} = \dfrac{V_1}{I_1} = j\omega L + 4R$

$Z_{12} = \dfrac{V_2}{I_1} = 4R$

② porta 1 a vuoto




$I_1 = 0$

Il generatore comandato si annulla poichè si annulla la corrente pilota

⇓

aperto.

$V_2 = R\, I_2$

$V_1 = V_2 = R I_2$

$Z_{12} = \dfrac{V_2}{I_2} = R$

$Z_{21} = \dfrac{V_1}{I_2} = R$

$$Z = \begin{bmatrix} 4R + j\omega L & 4R \\ R & R \end{bmatrix}$$

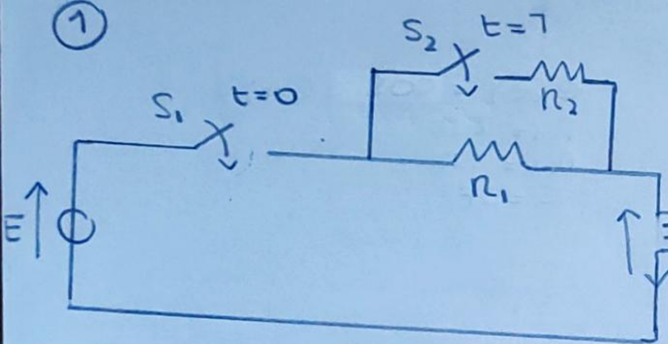
↓

matrice non simmetrica, doppio bipolo non reciproco.

# TUTORIAL 11

## perturbato

(1)




$E = 10V$
$R_1 = 1\Omega$
$R_2 = 2\Omega$
$L = 1H$

$I_L(0-) = I_L(0+)$

$0 < t < T$

$E = R_1 i + L\,Di$ → TENSIONE INDUTTIVA

$\tau = 1s$

$\alpha = -1Hz$

$I_L(0-) = I_L(0^+) = 0A$
(interruttore $S_1$ aperto)

$$\boxed{\tau = \frac{L}{R_{eq}}}$$

↓
Resistenza ai capi dell'induttore, spegnendo il generatore E

$$\boxed{\alpha = -\frac{1}{\tau} \qquad \tau = -\frac{1}{\alpha}}$$

$i(\infty) = \frac{E}{R_1}$ e' l'infinito riferito al tempo $0<t<T$

$I(t) = Ke^{-t/\tau} + I(\infty)$

$K = -10$

$I(t) = -10e^{t/\tau} + \frac{E}{R_1}$ con $0<t<T$

$T < t$

$E = R_{eq} \cdot i + Di \cdot L$ con $R_{eq} = \left(\frac{1}{R_2} + \frac{1}{R_1}\right)^{-1}$

$$\tau = \frac{L}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}}$$

$\tau = 1,5s$

$\alpha = -0,67\ s^{-1}\ (Hz)$

$\alpha < 0$ => la perturbazione si esaurisce

condizione iniziale del 2° transitorio => $i(T-) = i(T+)$

$i(T+) = i(T-)$
↓
ricavo dal primo transitorio valutata in $t=T$

$$V_c(0^-) = V_c(0^+)$$

$$V_c(T^-) = V_c(T^+)$$

$$I_c(0^-) = I_c(\infty) = 0A$$

$$I_L(0^+) = I_L(0^-)$$

$$I_L(T^-) = I_L(T^+)$$

$$V_L(0^-) = V_L(\infty) = 0V$$

trovo il nuovo K

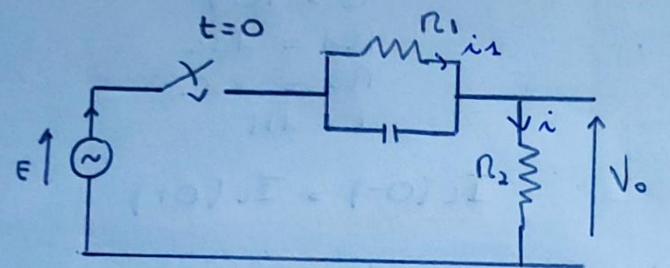
$$I(t) = K e^{-t/\tau} + I(\infty)$$

↳ riferita al 2° transitorio

↓

uso $I(T^-)$ per trovare il nuovo K

(2)




$e(t) = 10\cos(2t)\,V$

$R_1 = 2\,\Omega$

$R_2 = 1\,\Omega$

$C = 2F$

$V_C(0^-) = 2V$ e' data.

$V_0 = R_2 i$

$V_C = R_1 i_1$

$i_C = C \cdot DV_C$

$E = V_C + R_2 i$

$i = C DV + \dfrac{V_C}{R_1}$

$V_C(0^-) = V_C(0^+)$

$E = V_C + R_2 C DV_C + \dfrac{R_2}{R_1} V_C$

$R_2 C DV_C + \left(1 + \dfrac{R_2}{R_1}\right) V_C = E$

eq. omogenea: $R_2 C DV_C + \dfrac{R_1 + R_2}{R_1} V_C = 0$

$R_2 C \alpha + \dfrac{R_1 + R_2}{R_1} = 0$

$\alpha = -\dfrac{R_1 + R_2}{R_1 R_2 C}$

$\tau = 1/\alpha$

$V_C(t) = K e^{t/\tau} + V_C(\infty)$

CONSIDERO $V_0$

$V_0 = R_2 \cdot i$

$e = V_C + V_0 \quad \Rightarrow \quad V_C = e - V_0$

↳ sostituisco nella 3° equazione

$\dfrac{V_0}{R_2} = C DV_C + \dfrac{V_C}{R_1}$

$V_0(t) = K' e^{\alpha t} + V_0(\infty)$

$V_0(0^+) \neq V_0(0^-)$
$\quad$ 0

sostituisco t=0

$V_0(0^+) = e\cos(0) - V_C(0^+)$ → conosco