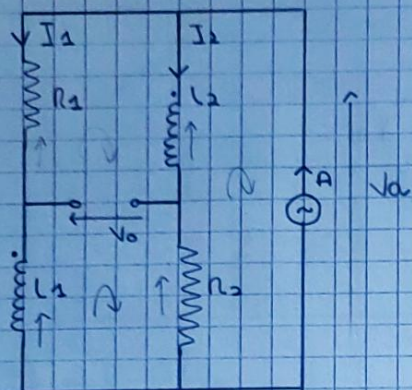


ESERCIZIO 2



$$\bar{A} = 10 \angle 45^\circ \text{ A}$$

$$R_1 = 4 \, \Omega$$

$$R_2 = 2 \, \Omega$$

$$L_1 = 10 \text{ mH}$$

$$L_2 = 30 \text{ mH}$$

$$\omega = 250 \text{ rad/s}$$

$$Z_1 = 2,5j \, \Omega$$

$$Z_2 = 7,5j \, \Omega$$

considero tutte le impedenze
per usare il derivatore di corrente

$$I_1 = A \cdot \frac{Z_2 + R_2}{R_1 + R_2 + Z_1 + Z_2}$$

$$I_1 = 6,65 \angle 61,03^\circ \text{ A}$$

$$I_2 = A - I_1 \quad I_2 = 4,04 \angle 17,97^\circ \text{ A}$$

$$V_A = (Z_2 + R_2) I_2$$

$$V_A = 31,4 \angle 93,04^\circ \text{ V}$$

$$V_0 = -R_2 I_2 + Z_1 I_1$$

$$V_0 = 22,92 \angle 165,96^\circ \text{ V}$$

$$Z_{eq} = \left(\frac{1}{Z_{L1} + R_1} + \frac{1}{Z_{L2} + R_2} \right)^{-1}$$

$$Z_{eq} = 3,14 \angle 48,04^\circ \, \Omega$$

$$S_A = V_A \cdot A^*$$

$$S_A = 209,94 + j233,5 \text{ VA}$$

$$P_A = 209,94 \text{ W}$$

$$Q_A = 233,5 \text{ VAR}$$

$$\varphi = \tan^{-1} \left(\frac{P_A}{Q_A} \right) \quad \cos \varphi = 0,67$$

$$\frac{\bar{V}_0}{\bar{A}} = 2,29 \angle 120,96^\circ \text{ V/A}$$

• CASO con $M = 15 \text{ mH} \Rightarrow$ induttori accoppiati

$$V_A = V_{L1} + V_2$$

$$V_{L1} = j\omega L_1 I_1 + j\omega M I_2$$

$$V_0 = V_{L2} - V_1$$

$$V_{L2} = j\omega M I_1 + j\omega L_2 I_2$$

$$V_0 = V_{L1} - V_2$$

$$A = I_1 + I_2$$

$$-R_1 I_1 - j\omega M I_1 + j\omega L_2 (A - I_1) = j\omega L_1 I_1 + j\omega M (A - I_1) - R_2 (A - I_1)$$

$$j\omega L_2 A - j\omega L_2 I_1 - R_1 I_1 + j\omega M I_1 = -R_2 A + R_2 I_1 + j\omega M A - j\omega L_1 I_1 + j\omega L_2 I_2 - j\omega M I_1$$

$$I_1 = \frac{A [j\omega L_2 - j\omega M + R_2]}{R_1 + j\omega L_2 + j\omega L_1 + R_2 - 2j\omega M}$$

$$I_1 = 6,54 \angle 84,31^\circ \text{ A}$$

$$-I_1 + A = I_2 \quad I_2 = 6,45 \angle 5,03^\circ \text{ A}$$

$$V_0 = 39,74 \angle 141,8^\circ \text{ V}$$

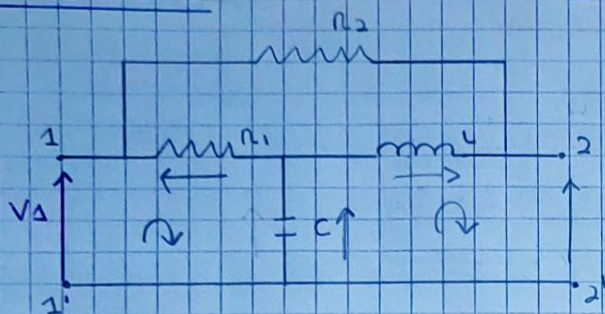
$$V_A = 54,09 \angle 106,97^\circ \text{ V}$$

$$Z_{eq} = \frac{V_A}{A}$$

$$Z_{eq} = 5,41 \angle 61,97^\circ \Omega$$

ESERCIZIO 2

11/07/2013



$$\begin{aligned} n_1 &= 4 \\ n_2 &= 3 \\ L &= 40 \text{ mH} \\ C &= 4 \text{ mF} \\ \omega &= 100 \text{ rad/s} \end{aligned}$$

$$\begin{aligned} Z_1 &= 4 \Omega \\ Z_2 &= 5 \Omega \\ Z_C &= -2.5j \Omega \\ Z_L &= 4j \Omega \end{aligned}$$

$$Z_{11'} = \left[\frac{1}{n_2 + L} + \frac{1}{n_1} \right]^{-1} + Z_C$$

• considero la porta 2-2' un ~~aperto~~, dove non passa corrente.

$$Z_{11'} = 2.52 - 1.84j \Omega$$

$$Z_{22'} = \left[\frac{1}{n_1 + n_2} + \frac{1}{Z_L} \right]^{-1} + Z_C$$

• considero la porta 1-1' un ~~aperto~~, dove non passa corrente.

$$Z_{22'} = 1.48 + 0.84j \Omega$$

$$Z_{12} = Z_{21} = \left(\frac{V_1}{I_2} \right)_{I_1=0}$$

$I_2 = 1 \text{ A} \rightarrow$ corrente a mio piacimento, particolare, KVL

$$\textcircled{2} V_1 = n_1 I_1 + Z_C I_C \quad I_C = I_2 = 1 \text{ A}$$

$$\textcircled{1} I_1 = 1 \cdot \frac{j\omega L}{n_1 + n_2 + j\omega L} = 1 \cdot \frac{4j}{9 + 4j} = 0.406 \angle 66.04^\circ \text{ A}$$

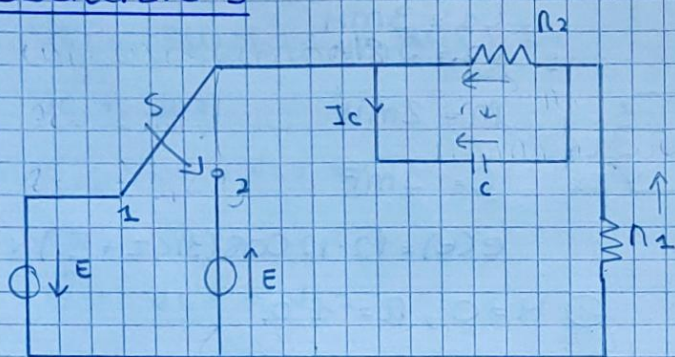
$$Z_{12} = Z_{21} = \left(\frac{V_1}{I_2} \right)_{I_1=0} = 0.66 - 1.01j \Omega$$

$$Z_{12} = 4 \left[\frac{j\omega L}{9 + j\omega L} \right] - \frac{j}{\omega C}$$

$$\begin{aligned} \omega = 0 &\Rightarrow Z_{12} = \infty \\ \omega = \infty &\Rightarrow Z_{12} = 4 \end{aligned}$$

22.06.2007

ESERCIZIO 3



$$\begin{aligned} E &= 12V \\ C &= 20 \text{ mF} \\ R_2 &= 4 \Omega \\ R_1 &= 6 \Omega \end{aligned}$$

$$t = 0^-$$

$$I_e = \frac{E}{R_{eq}} \quad R_{eq} = R_1 + R_2 = 10 \Omega$$

$$I_e = 1,2 \text{ A}$$

$$V_c = -R_2 I_e$$

$$V_c(0^-) = V_c(0^+) = -4,8 \text{ V}$$

$$I_c(0^-) = 0 \text{ A}$$

$$V_c(t) = K_1 e^{\alpha t} + K_2$$

$$K_2 = V(\infty) = 4,8 \text{ V}$$

$$K_1 = -9,6 \text{ V}$$

$$t = \infty$$

$$I_c(\infty) = 0 \text{ A}$$

$$I_e = \frac{E}{R_{eq}} \quad I_e = 1,2$$

$$V_c(\infty) = V_2$$

$$V_c(\infty) = 4,8 \text{ V}$$

$$I_e = I_1 \quad I_e = I_c + I_2$$

$$E - R_2 I_2 - R_1 I_1 = 0$$

$$E - R_2 (I_e - I_c) = R_1 I_e$$

$$E + R_2 I_c = (R_1 + R_2) I_e$$

$$V_c = R_2 I_2$$

$$I_e = \frac{12 + 4 I_c}{10}$$

$$V_c = R_2 (I_e - I_c) \quad V_c = 4 \left[\frac{12}{10} + \frac{4}{10} I_c - I_c \right]$$

$$I_c(0^+) = - \left(-\frac{6}{5} - \frac{12}{10} \right) \frac{5}{3} = 4 \text{ A}$$

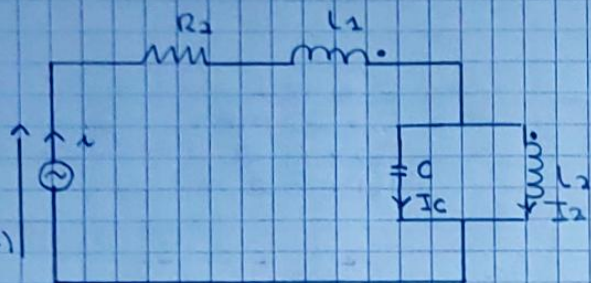
$$\tau = -\frac{1}{\alpha}$$

$$\tau = R_{eq} \cdot C$$

$$\alpha = -\frac{1}{R_{eq} \cdot C}$$

$$\alpha = -20,83 \text{ Hz}$$

17. 07. 2007



$$L_1 = 3 \text{ mH}$$

$$L_2 = 5 \text{ mH}$$

$$M = 2 \text{ mH}$$

$$C = 1 \text{ mF}$$

$$e(t) = \sqrt{2} \cdot 12 \cos(314t + \frac{\pi}{2}) \text{ V}$$

$$\omega = 314 \text{ rad/s}$$

$$\textcircled{1} M = 0, n = 1$$

$$Z_C = -3,18 j \Omega$$

$$Z_{L1} = 0,942 j \Omega$$

$$Z_{L2} = 1,57 j \Omega$$

$$\bar{I} = \frac{\bar{E}}{Z_{eq}}$$

$$Z_{eq} = R_2 + Z_{L1} + \left(\frac{1}{Z_C} + \frac{1}{Z_{L2}} \right)^{-1}$$

$$\bar{I} = 2,47 - 1,49 j \text{ A}$$

$$\bar{I} = 2,88 \angle -31,10^\circ$$

$$\bar{I}_C = \bar{I} \cdot \frac{Z_{L2}}{Z_{L2} + Z_C}$$

$$\bar{I}_C = -2,4 + 1,45 j \text{ A}$$

$$\bar{I}_C = 2,8 \angle 149^\circ \text{ A}$$

$$\bar{I}_2 = \bar{I} - \bar{I}_C$$

$$\bar{I}_2 = 4,87 - 2,9 j \text{ A}$$

$$\bar{I}_2 = 5,7 \angle -31,1^\circ \text{ A}$$

$$\omega_{ris} = \frac{1}{\sqrt{LC}}$$

→ solo se C e L sono in parallelo

$$\omega_{risp} = 447,2 \text{ rad/s}$$

$$Z_{eq} = 1 + j\omega L_1 + \left[\frac{1}{-\frac{j}{\omega C}} + \frac{1}{j\omega L_2} \right]^{-1}$$

$$\omega = 0 \quad Z_{eq} = 1$$

$$\omega = \infty \quad Z_{eq} = \infty$$

Caso $M = 2MH$, $n = 0$

$$V_{L1} = j\omega L_1 I_1 + j\omega M I_2$$

$$V_{L2} = j\omega M I_1 + j\omega L_2 I_2$$

$$V_C = V_{L2}$$

$$E - V_{L1} - V_{L2} = 0$$

$$I_e = I_1$$

$$I_1 = I_C + I_2$$

$$I_2 = I_1 - I_C$$

$$E = V_{L1} + V_{L2}$$

$$E = j\omega L_1 I_1 + j\omega M(I_1 - I_C) + j\omega M I_1 + j\omega L_2(I_1 - I_C)$$

$$E = j\omega L_1 I_1 + j\omega M I_1 - j\omega M I_C + j\omega M I_1 + j\omega L_2 I_1 - j\omega L_2 I_C$$

$$E + j\omega M I_C + j\omega L_2 I_C = I_1 (j\omega L_1 + j\omega M + j\omega M + j\omega L_2)$$

$$I_1 = \frac{E + j\omega I_C + j\omega L_2 I_C}{j\omega L_1 + 2j\omega M + j\omega L_2}$$

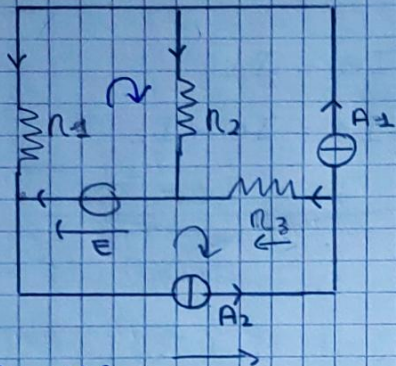
$$I_2 = \frac{E + j\omega I_C + j\omega L_2 I_C}{j\omega L_1 + 2j\omega M + j\omega L_2} - I_C$$

$$-\frac{j}{\omega C} I_C = j\omega M \left[\frac{E + j\omega I_C + j\omega L_2 I_C}{j\omega L_1 + 2j\omega M + j\omega L_2} \right] +$$

$$j\omega L_2 \left[\frac{E + j\omega I_C + j\omega L_2 I_C}{j\omega L_1 + 2j\omega M + j\omega L_2} \right] - j\omega L_2 I_C$$

10.02.2012

ESERCIZIO 2



$$A_1 = 5 \text{ A}$$

$$E = 12 \text{ V}$$

$$R_1 = 3 \, \Omega$$

$$R_2 = 2 \, \Omega$$

$$R_3 = 1 \, \Omega$$

$$A_2 = 7 \text{ A}$$

$$I' = A_1 \cdot \frac{R_2}{R_1 + R_2} = 2 \text{ A}$$

$$I_e^1 = I'$$

$$I^2 = 0 \text{ A}$$

$$I_e^2 = 7 \text{ A}$$

$$I^E = -\frac{E}{R_{eq}}$$

$$(I^E = -I_e^E)$$

$$I^E = -2,4 \text{ A}$$

$$I_e^{TOT} = -7,4 \text{ A}$$

$$P_E = 88,8 \text{ generatore}$$

$$P_1 = R_1 \cdot I_{TOT}^2$$

$$P_1 = 0,48 \text{ utizzatore}$$

$$A_2 = -3 \text{ I}$$

effetto di A_1 :

$$R_1 I_1 = R_2 (A - I_1)$$

$$I_3^{A_1} = 5 \text{ A}$$

$$I_1 (R_1 + R_2) = A \cdot R_2$$

$$I = A \frac{R_2}{R_1 + R_2} \quad I = 2 \text{ A}$$

effetto E :

$$E + R_1 I = -R_2 I$$

$$I_2 = -I$$

$$I_2 = -5 \text{ I} \quad I = -2,4 \text{ A}$$

$$P_{R1} = 0,48 \text{ W}$$

$$P_{A2} = V_A \cdot A_2$$

$$I_3^{TOT} = I_A^3 - I^3 E = 3,8 \text{ A}$$

$$V_3 = 3,8 \text{ V}$$

$$E + V_{A2} = -V_3$$

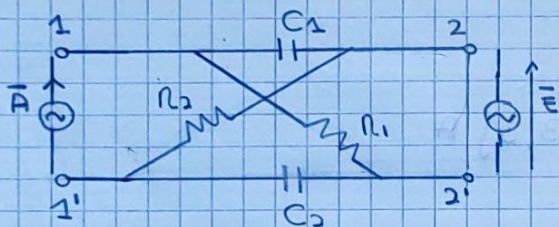
$$V_{A2} = 15,8 \text{ V}$$

$$P_{A2} = 18,96 \text{ W}$$

USO LA I_3 TOTALE
per trovare V_3 .
USO LA I_{TOT} per
trovare A_2

! ATTENZIONE VERSO I_3 !

ESENCIZIO 1



$$Z_{C1} = -1j$$

$$Z_{C2} = -0,5j$$

$$Z_{eq}^{11'} = \left(\frac{1}{R_1 + C_2} + \frac{1}{R_2 + C_1} \right)^{-1}$$

$$Z_{eq}^{11'} = 0,67 - 0,33j \, \Omega$$

$$Z_{eq}^{22'} = \left(\frac{1}{R_1 + Z_{C1}} + \frac{1}{R_2 + Z_{C2}} \right)^{-1}$$

$$Z_{eq}^{22'} = 0,73 - 0,47j \, \Omega$$

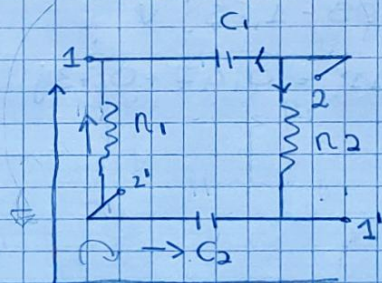
$$R_1 = 1 \, \Omega$$

$$R_2 = 2 \, \Omega$$

$$C_1 = 10 \text{ mF}$$

$$C_2 = 20 \text{ mF}$$

$$\omega = 100 \text{ rad/s}$$



$$A = 10 \text{ A}$$

$$I_1 = A \cdot \frac{R_2 + Z_{C2}}{R_1 + R_2 + Z_{C1} + Z_{C2}}$$

$$I_1 = 0,6 + 0,13j \text{ A}$$

$$I_2 = 0,4 - 0,13j \text{ A}$$

$$V_1 = V_{R1} - V_{C2}$$

$$V_1 = 0,67 + 0,33j \text{ V}$$

$$Z_{12} = Z_{21} = 0,67 + 0,33j \, \Omega$$

$$\bar{Z} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$\bar{Y} = \frac{1}{\bar{Z}} = \frac{1}{\det \bar{Z}} \begin{bmatrix} Z_{22} & -Z_{21} \\ -Z_{12} & Z_{11} \end{bmatrix}$$

$$\det = -6 \cdot 10^{-3} - 0,998j$$

$$Y_{11} = 0,47 + 0,73j$$

$$Y_{12} = Y_{21} = 0,33 - 0,67j$$

$$Y_{22} = 0,33 + 0,67j$$

$$Z_{TH} = Z_{22'}$$

$$Z_{TH} = 0,73 - 0,47j \, \Omega$$

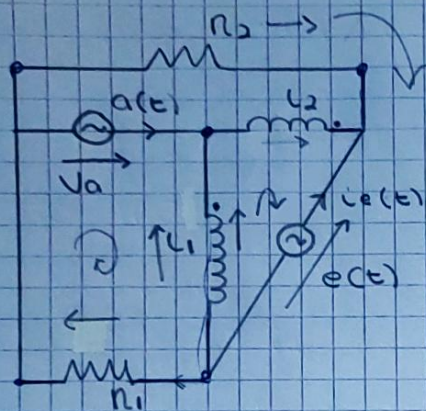
$$V_{TH} = V_{N1} - V_{C1}$$

$$I_N = A \cdot \frac{Z_{C1} + n_2}{n_1 + n_2 + Z_{C1} + Z_{C2}} = 3,3 - 3,3j \, A$$

$$I_C = 1,65 - 1,65j \, A$$

$$V_{TH} = 4,95 - 1,65j \, V$$

$$Y_{NO} = Y_{11'} = 0,47 + 0,73j$$



$M=0$

effetto A:

$Z_{L1} = 876j$

$Z_{L2} = 879j$

$Z_{eq} = \left(\frac{1}{Z_{L1}} + \frac{1}{Z_{L2}} \right)^{-1} + \left(\frac{1}{R_1} + \frac{1}{R_2} \right)^{-1}$

$Z_{eq} = 3882,3529 + 438,7487j$

$V_a = Z_{eq} \cdot A \quad V_a = 6,11 - 4,87j \text{ V}$

$I_{L2} + I_e = I_{n2}$

$I_{n2} = A \cdot \frac{R_1}{R_1 + R_2}$

$I_{n2} = 7,05 \cdot 10^{-4} \angle -45^\circ \text{ A}$

$I_{L2} = A \cdot \frac{Z_{L1}}{Z_{L1} + Z_{L2}}$

$I_{L2} = 9,98 \cdot 10^{-4} \angle -45^\circ \text{ A}$

$I_e = I_{n2} - I_{L2}$

$I_e = -2,07 \cdot 10^{-4} + 2,07 \cdot 10^{-4}j \text{ A}$

effetto di E:

$Z_1 = 584j$

$Z_2 = 586j$

$Z_{eq} = \left(\frac{1}{Z_1 + Z_2} + \frac{1}{R_1 + R_2} \right)^{-1}$

$Z_{eq} = 80,1439 + 1164,4842j \Omega$

$I_e = \frac{E}{Z_{eq}}$

$I_e = -5,6461 \cdot 10^{-3} - 0,011544j \text{ A}$

$$I_n = I_e \cdot \frac{Z_2 + Z_1}{Z_2 + Z_1 + R_1 + R_2}$$

$$I_n = 8,8 \cdot 10^{-4} \angle -30^\circ \text{ A}$$

$$I_L = 0,0128 \angle -120^\circ \text{ A}$$

$$V_A - V_{L1} + V_{R1} = 0$$

$$V_A = V_{L1} - V_{R1}$$

$$V_A = -7,487 \angle -30^\circ - 8,8 \cdot 10^{-4} \angle -30^\circ$$

$$V_A = 1,9 - 1,0976j \text{ V}$$

$$I_e^{\text{TOT}} = 0,01278 \angle -117,57^\circ \text{ A}$$

$$P_E = 8,12784 \cdot 10^{-3} \text{ W}$$

$$Q_E = 0,1915276 \text{ Var}$$

$$P_A = 0,01976 \text{ W}$$

$$Q_A = 2,8883 \cdot 10^{-3} \text{ Var}$$

$$M = 80 \cdot 10^{-6} \text{ H}$$

$$\text{effetto di A: } A = I_1 + I_2$$

$$V_{L2} = V_{L1}$$

$$V_{L1} = j\omega L_1 I_1 - j\omega M I_2$$

$$V_A - V_{L1} - V_{R1} = 0$$

$$V_{L2} = j\omega M I_1 + j\omega L_2 I_2$$

$$j\omega L_1 I_1 - j\omega M I_2 = j\omega L_2 I_2 - j\omega M I_1$$

$$I_1 [L_1 + M] = I_2 [M + L_2]$$

$$I_1 = I_2 \left[\frac{M + L_2}{M + L_1} \right]$$

$$I_2 \left[\frac{M + L_2}{M + L_1} \right] + I_2 = A$$

$$I_2 = 9,98 \cdot 10^{-4} \angle -45^\circ \text{ A}$$

$$I_1 = 10^{-3} \angle -45^\circ \text{ A}$$

$$V_1 = R_1 I_{n1} \quad V_2 = R_2 I_{n2}$$

$$A = I_{n1} + I_{n2}$$

$$R_1 I_1 = R_2 (A - I_{n1})$$

$$(R_1 + R_2) I_1 = R_2 A$$

$$V_A = V_{R1} + V_{L1}$$

$$I_{n1} = \frac{11000}{17000} \text{ A}$$

$$V_A = 5,94 - 5,0397j \text{ V}$$

$$I_{n1} = 1,29 \cdot 10^{-3} \angle -45^\circ$$

$$I_e = I_{L1} - I_{M1}$$

$$I_e = -2,05 \cdot 10^{-4} + 2,05 \cdot 10^{-4} j \text{ A}$$

$$Z_{eq} = \frac{V_a}{A}$$

$$Z_{eq} = 3880 + 318,75 j \Omega$$

- effetto di E

$$E = V_{L1} + V_{L2}$$

$$I_1 = I_2$$

$$V_{L1} = j\omega L_1 I_1 + j\omega M I_2$$

$$V_{L2} = j\omega L_2 I_2 + j\omega M I_1$$

$$E = (j\omega L_1 + 2j\omega M + j\omega L_2) I_1$$

$$j\omega M = 160 j$$

$$I_1 = \frac{E}{j\omega L_1 + 2j\omega M + j\omega L_2}$$

$$I_1 = \frac{15 \angle -30^\circ}{1490 j} = 0,010067 \angle -120^\circ \text{ A}$$

$$E = n_1 I_{n1} + n_2 I_{n2}$$

$$I_{n1} = I_{n2} = \frac{E}{n_1 + n_2}$$

$$I_{n1} = I_{n2} = 8,8 \cdot 10^{-4} \angle -30^\circ \text{ A}$$

$$I_e = I_{n1} + I_{n2}$$

$$I_e = -4,269 \cdot 10^{-3} - 9,159 \cdot 10^{-3} j \text{ A}$$

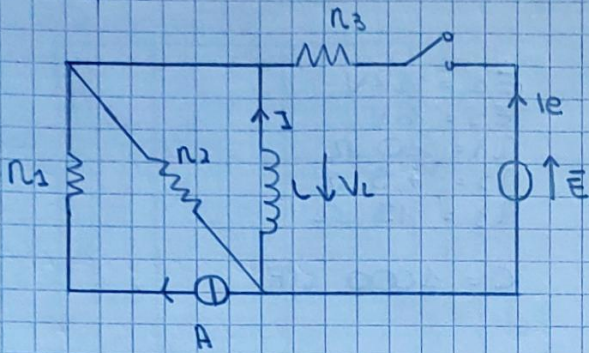
$$Z_{eq} = \frac{E}{I_e}$$

$$Z_{eq} = 129,60 + 1478,65 j \Omega$$

$$V_A = V_{L1} - V_{n1}$$

$$V_A = 1,9052 - 1,1 j \text{ V}$$

24/06/2004



$$A = 7 \text{ A} \quad E = 9 \text{ V}$$

$$R_1 = 4 \Omega \quad R_2 = 8 \Omega$$

$$R_3 = 2 \Omega$$

$$L = 21 \text{ mH}$$

$t = 0$ l'interruttore si chiude

$$I_e(0^-) = 0 \text{ A}$$

$$I(0^-) = -7 \text{ A}$$

$$V(0^-) = 0 \text{ V}$$

$$I(0^-) = I(0^+) = -7 \text{ A}$$

$$V(\infty) = 0 \text{ V}$$

$$\tau = \frac{L}{R_{eq}}$$

$$\tau = 13,1 \text{ ms}$$

$$I_e(0^+) = \frac{E}{R_{eq}}$$

$$I_e(0^+) = 0,9 \text{ A}$$

$$V(0^+) = -E + V_L$$

$$V(0^+) = -7,2 \text{ V}$$

$$I(\infty) = I^A(\infty) + I^E(\infty)$$

$$I_e(\infty) = \frac{E}{R_3} = 4,5 \text{ A}$$

$$I(\infty) = -4,5 \text{ A}$$

$$I^A(\infty) = 7 \text{ A}$$

$$I^A(\infty) = -7 \text{ A}$$

$$I(\infty) = -11,5 \text{ A}$$

$$I(\tau) = K e^{-t/\tau} + I(\infty)$$

$$-7 = K - 11,5$$

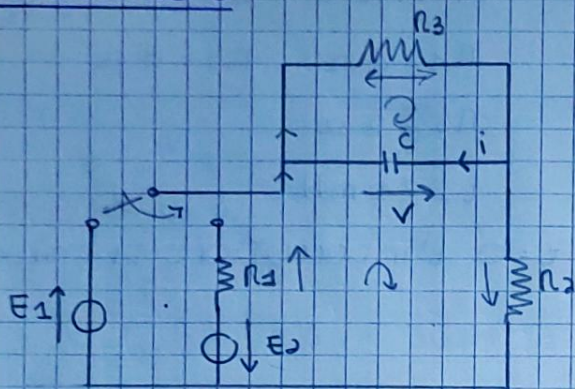
$$K = 4,5$$

$$I(2\tau) = 4,5 e^{-2} - 11,5$$

$$I(2\tau) = -10,9 \text{ A}$$

26/06/2003

ESENCIO 3



$$E_1 = 12V$$

$$E_2 = 6V$$

$$R_1 = 10\Omega$$

$$R_2 = 7,5\Omega$$

$$R_3 = 15\Omega$$

$$C = 1000\mu F$$

$$I_e(0^-) = \frac{E}{R_3 + R_2} = 0,533 A$$

$$V = -R_3 I_e(0^-) \quad V(0^-) = -8V$$

$$I_3(0^+) = \frac{V}{R_3} = -0,533 A$$

$$V + V_2 - E_2 + V_1 = 0$$

$$-\frac{V - E}{R_2 + R_1} = I_e \quad I_e = 0,8 A$$

$$I_e = I + I_3$$

$$I = 1,33 A$$

$$\tau_1 = C \cdot R_{eq}$$

$$R_{eq} = \left(\frac{1}{R_2} + \frac{1}{R_1} \right)^{-1}$$

$$\tau = 8,1 ms$$

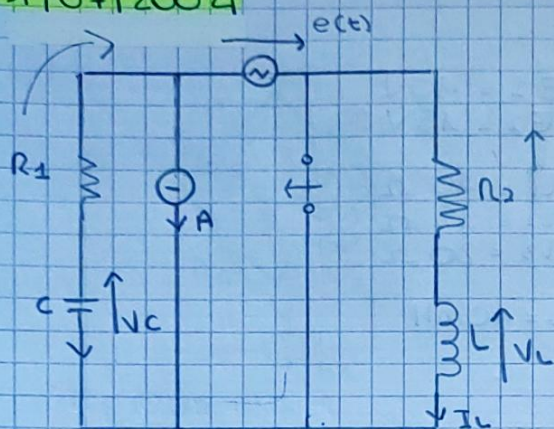
$$\tau_2 = C \cdot R_{eq}$$

$$R_{eq} = \left(\frac{1}{R_3} + \frac{1}{R_2} \right)^{-1}$$

$$\tau = 5 \cdot 10^{-3} s$$

$$V(\infty) = -8 V$$

11/07/2004



$$A = 2 \text{ mA}$$

$$R_1 = 5 \text{ k}\Omega$$

$$R_2 = 3 \text{ k}\Omega$$

$$L = 15 \text{ mH}$$

$$C = 0,02 \text{ }\mu\text{F}$$

$$e(t) = 12\sqrt{2} \cos(10^5 t + \pi/6) \text{ V}$$

$$I_L(t) = -2 \text{ mA}$$

$$V_2(t) = -R_2 \cdot A = -6 \text{ V}$$

$$I_C(t) = 0 \text{ A}$$

$$V_C(t) = V_2(t) = -6 \text{ V}$$

$$V_L(t) = 0 \text{ V}$$

$$j\omega L = 1500j \quad j\omega C = -500j$$

$$I_L = I_C = I_e$$

$$I_e = \frac{\bar{E}}{R_1 + R_2 + Z_L + Z_C} = 1,49 \cdot 10^{-3} \angle 22,87^\circ \text{ A}$$

$$I_L = I_C = 1,49 \sqrt{2} \cos(10^5 t + 22,87^\circ) \text{ mA}$$

$$V_C = Z_C \cdot I_C$$

$$V_C = 0,74 \sqrt{2} \cos(10^5 t - 112,87^\circ)$$

$$V_L = I_L \cdot Z_L$$

$$V_L = 2,23 \sqrt{2} \cos(10^5 t + 112,87^\circ)$$

$$I_L(0^-) = 0 \text{ A}$$

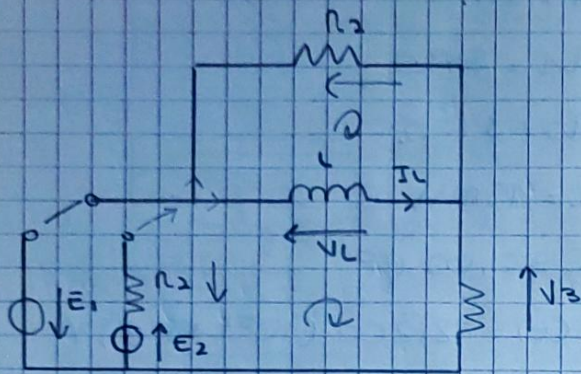
$$I_C(0^+) = -2 \text{ A}$$

$$I_L(0^+) = 0 \text{ A}$$

$$V_L(0^-) = 0 \text{ V}$$

20106 / 2005

③



$$E_1 = 40V$$

$$E_2 = 15V$$

$$R_1 = 10 \Omega$$

$$R_2 = 15 \Omega$$

$$R_3 = 20 \Omega$$

$$L = 10mH$$

$$I(\infty) = I_{e_2} = \frac{E}{R_2 + R_3}$$

$$I(\infty) = \frac{15}{30} = 0,5A$$

$$V_1(\infty) = 0V$$

$$V_3(\infty) = I(\infty) \cdot R_3$$

$$V_3(\infty) = \frac{1}{2} \cdot 20 = 10V$$

$$I_L(0-) = I_L(0+)$$

$$I_L(0-) = -\frac{E_1}{R_3} = -0,5A \quad I_L(0+) = -0,5A$$

$$\tau = \frac{L}{R_{eq}} \quad \tau = \frac{10 \cdot 10^{-3}}{10} = 1ms$$

$$E_2 - R_1 I_e(0+) - V_L - R_3 I_e(0+) = 0$$

$$R_2 (I_e - I_L(0+)) = V_L$$

$$R_2 I_e + \frac{R_2}{2} = V_L$$

$$15 - 10 I_e(0+) - 15 I_e(0+) - 7,5 - 20 I_e(0+) = 0$$

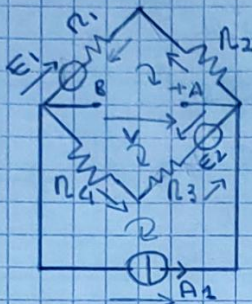
$$I_e(0+) = 0,1667A$$

$$V_L(0+) = 10V$$

$$V_3(0+) = 3,3V$$

04/05/2001

②



$$E_1 = 12V$$

$$E_2 = 20V$$

$$A_1 = 1,2A$$

$$R_1 = 1\Omega$$

$$R_2 = 5\Omega$$

$$R_3 = 2\Omega$$

$$R_4 = 4\Omega$$

• effetto E_1

$$I_e = \frac{E_1}{R_{eq}} \quad I_e = \frac{12}{12} = 1A$$

$$E - V_1 - V_2 = V \quad V = 6V$$

• effetto E_2

$$I_e = \frac{20}{12} = 1,67A$$

$$V + E_2 = V_3 + V_4$$

$$V = V_3 + V_4 - E_2 \quad V = -10V$$

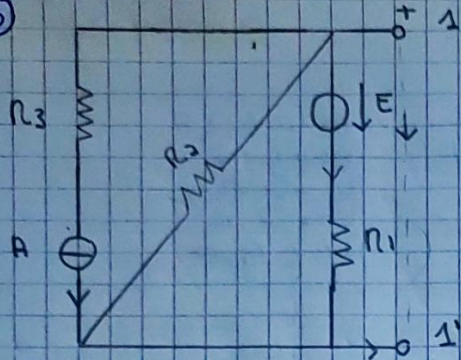
• effetto di A_1 :

$$I_1 = A_1 \cdot \frac{6}{12} = 0,6A$$

$$V = V_3 + V_4 \quad V = 3,6V$$

$$P_{A1} = 0,48W$$

③



$$E = 1,5 \text{ V}$$

$$A = 0,15 \text{ A}$$

$$R_1 = 200 \, \Omega$$

$$R_2 = 300 \, \Omega$$

$$R_3 = 500 \, \Omega$$

$$R_{TH} = \left(\frac{1}{R_1} + \frac{1}{R_2} \right)^{-1}$$

$$R_{TH} = 120 \, \Omega$$

$$I_{CC} = I_E + I_A$$

$$I_{CC} = 7,5 \cdot 10^{-3} + 0,15$$

$$I_{CC} = 0,1575 \text{ A}$$

$$V_{TH} = -R_{TH} \cdot I_{CC}$$

$$V_{TH} = -18,9$$

$$I_{CC} = -I_{NO}$$

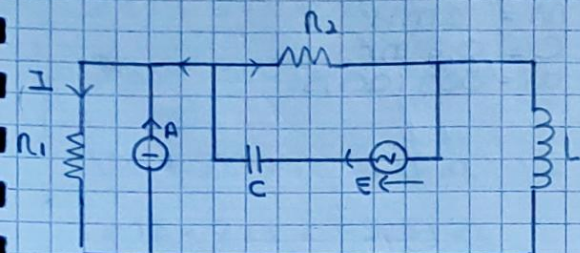
$$I_{NO} = -0,16 \text{ A}$$

$$G_{NO} = \frac{1}{R_{TH}}$$

$$G_{NO} = 8,3 \cdot 10^{-3} \text{ S}$$

26/06/2008

ESERCIZIO 1



$$R_1 = 12 \, \Omega$$

$$A = 2 \text{ A}$$

$$R_2 = 30 \, \Omega$$

$$L = 4 \text{ mH}$$

$$C = 1,2 \text{ mF}$$

$$\omega = 200 \text{ rad/s}$$

$$e = 5\sqrt{2} \cos(\omega t + \frac{\pi}{3}) \text{ V}$$

$$R_A = \left(\frac{1}{R_2} + \frac{1}{R_1} \right)^{-1}$$

$$R_A = 8,57 \, \Omega$$

$$I_A = A \cdot \frac{R_2}{R_1 + R_2}$$

$$I_A = 1,43 \text{ A}$$

$$Z_L = 0,8j \, \Omega$$

$$Z_C = -4,17j \, \Omega$$

$$Z_{eq} = \left(\frac{1}{R_2} + \frac{1}{R_1 + Z_L} \right)^{-1} + Z_C$$

$$Y_{eq} = \frac{1}{Z_{eq}} = 0,098 + 0,043j \, \Omega^{-1}$$

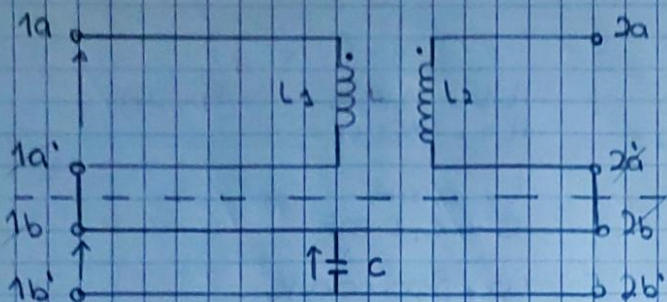
$$I_E = \frac{E}{Z_{eq}} = 0,534 \, \underline{83,68} \text{ A}$$

$$I_e = I_E \cdot \frac{R_2}{R_1 + R_2}$$

$$I_e = 0,38 \, \underline{83,68} \text{ A}$$

$$I_e = 0,38 \, \underline{0,46\pi} \text{ A}$$

ESERCIZIO 2



$$\begin{aligned} L_1 &= 12 \text{ mH} \\ L_2 &= 6 \text{ mH} \\ M &= 1 \text{ mH} \\ C &= 0,8 \text{ mF} \\ \omega &= 500 \text{ rad/s} \end{aligned}$$

$$M = 0$$

$$Z_{11} = j\omega L_1 = 6j \, \Omega$$

$$Z_{22} = j\omega L_2 = 3j \, \Omega$$

$$Z_A = \begin{bmatrix} 6j & 0 \\ 0 & 3j \end{bmatrix}$$

$$Z_{12} = Z_{21} = \left(\frac{V_1}{I_2} \right)_{I_1=0}$$

$$Z_{12} = Z_{21} = 0j \, \Omega$$

$$Z_{11} = -\frac{j}{\omega C} = -2,5j$$

$$Z_{22} = -j/\omega C = -2,5j$$

$$Z_{12} = Z_{21} = -j/\omega C = -2,5j$$

$$Z_B = \begin{bmatrix} -2,5j & -2,5j \\ -2,5j & -2,5j \end{bmatrix}$$

$$M = 1 \text{ mH}$$

$$Z_{12} = Z_{21} = \left(\frac{V_1}{I_2} \right)_{I_1=0}$$

$I_2 = 1 \text{ A}$
 I_2 passa tutta in C
 perché 11' è aperto.

$$V_1 = -j/\omega C$$

$$V_{L1} = V_1 = j\omega L_1 I_1 + j\omega M I_2$$

$$V_2 = V_{L2} = j\omega M I_1 + j\omega L_2 I_2$$

$$Z_{11} = 6j \, \Omega \quad Z_{22} = 3j \, \Omega$$

$$Z_{12} = Z_{21} = \left(\frac{V_1}{I_2} \right)_{I_1=0}$$

$$\text{Se } I_1 = 0:$$

$$V_1 = j\omega M I_2 = 0,5j \cdot I_2$$

$$Z_{12} = Z_{21} = \frac{0,5j \cdot I_2}{I_2} = 0,5j \, \Omega$$

$$Z_A = \begin{bmatrix} 6j & 0,5j \\ 0,5j & 3j \end{bmatrix}$$

Z_B non cambia perché non è presente M nel circuito.

• caso in cui diventa un circuito unico:

$$Z_{11}' = Z_{L1} + Z_C$$

$$Z_{22}' = Z_{L2} + Z_C$$

$$Z_{11}' = 3,5j \, \Omega$$

$$Z_{22}' = 0,5j \, \Omega$$

$$V_1 = Z_C = -2,5j \, \Omega$$

$$Z_{12} = Z_{21} = -2,5j \, \Omega$$

$$Z = \begin{bmatrix} 3,5j & -2,5j \\ -2,5j & 0,5j \end{bmatrix}$$

$$M = 1mH$$

$$Z_{11} = 3,5j$$

$$Z_{22} = 0,5j$$

$$V_1 = V_{L1} + (-j/\omega C)$$

$$V_1 = j\omega L_1 I_1 + j\omega M I_2 - j/\omega C \cdot I_2$$

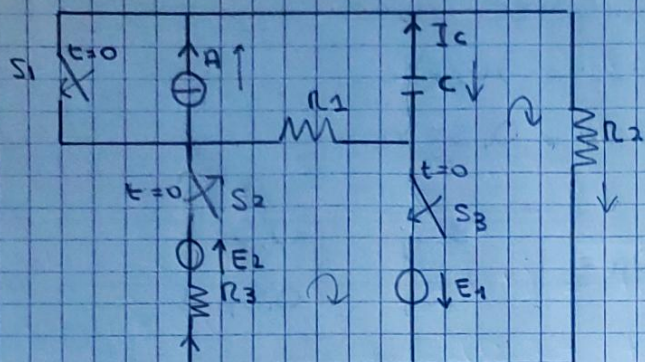
$I_1 = 0$

$$V_1 = j\omega M I_2 - 2,5j \quad \text{con } I_2 = 1A$$

$$V_1 = -2j$$

$$Z = \begin{bmatrix} 3,5j & -2j \\ -2j & 0,5j \end{bmatrix}$$

ESERCIZIO 3



$$\begin{aligned} R_1 &= 150 \, \Omega \\ R_2 &= 250 \, \Omega \\ R_3 &= 40 \, \Omega \\ C &= 7 \, \text{mF} \\ A &= 0,1 \, \text{A} \\ E_1 &= 12 \, \text{V} \\ E_2 &= 110 \, \text{V} \end{aligned}$$

$$I_C(0^-) = 0 \, \text{A}$$

$$V_C(0^-) = V_C(0^+)$$

$$V_C = 110 - V_2 - V_3$$

$$V_C(0^+) = V_C(0^-) = 81 \, \text{V}$$

$$V(\infty) = R_2 I_2 - E_1$$

$$I_2 = \frac{12}{400} = 0,03 \, \text{A}$$

$$V(\infty) = 7,5 - 12 = -4,5 \, \text{V}$$

$$\tau = C \cdot R_{eq} \quad \tau = 0,656 \, \text{s}$$

$$V(\tau) = K e^{-\tau/\tau} + V(\infty)$$

$$81 = K - 4,5$$

$$K = 85,5$$

$$V(\tau) = 85,5 e^{-\tau/\tau} + (-4,5)$$

$$V(\tau) = 26,95$$

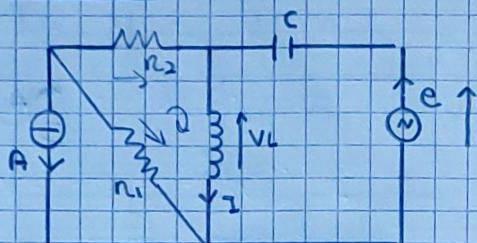
$$DV(0^+) = -\frac{85,5}{0,656} = -130,34 \, \text{V/s}$$

$$\underline{I(0^+) = C \cdot \dot{V}_C}$$

$$I(0^+) = -0,912 \, \text{A}$$

24/06/2004

ESENCIO 1



$$A = 20 \text{ A}$$

$$R_1 = 7 \, \Omega$$

$$L = 12 \text{ mH}$$

$$R_2 = 5 \, \Omega$$

$$C = 16 \, \mu\text{F}$$

$$e(t) = 15\sqrt{2} \cos(314t - \pi/6) \text{ V}$$

$$\bar{E} = 15 \sqrt{2} \text{ V}$$

$$Z_L = 3,768j \, \Omega$$

$$Z_C = -199,04j \, \Omega$$

A) $V_L(t) = 0 \text{ V}$

$$V + V_1 = V_2 \quad V = 0 \text{ V}$$

$$N_1 I_1 = N_2 I_2$$

$$N_1 I_1 = N_2 A - N_2 I_1$$

$$I_2 = A - I_1$$

$$I_1 = \frac{N_2 A}{12} = 8,33 \text{ A}$$

$$I_2 = A - I_1 = 11,67 \text{ A}$$

$$I_2 = -I \quad I = -11,67 \text{ A}$$

E) $Z_{eq} = Z_C \left[\frac{1}{Z_L} + \frac{1}{N_1 + N_2} \right]^{-1}$

$$I_e = \frac{E}{Z_{eq}}$$

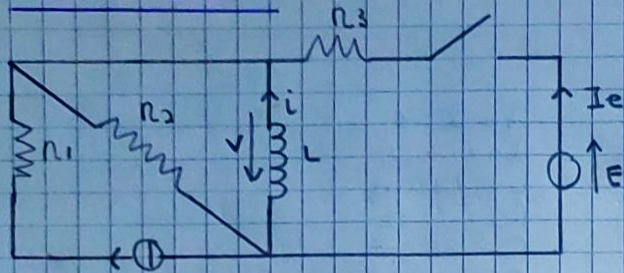
$$I = \frac{E}{Z_{eq}} \cdot \frac{N_1 + N_2}{N_1 + N_2 + Z_L}$$

$$I = 0,07 \sqrt{39.05} \text{ A}$$

$$V_L = Z_L \cdot I_L$$

$$V_L = 0,29 \sqrt{129.05} \text{ V}$$

ESERCIZIO 2



$$V(0^-) = 0V$$

$$I(0^+) = -7A$$

$$V(\infty) = 0V$$

$$I(0^-) = -7A$$

$$I_e(0^-) = 0A$$

$$I_e(0^+) = \frac{E}{R_{eq}}$$

$$I_e(0^+) = 0,9A$$

$$V + E = V_3$$

$$I_3^{TOT} = I_3^E + I_3^A$$

$$I_3^{TOT} = 0,9A$$

$$V = R_3 I_3 - E$$

$$V(0^+) = -7,2V$$

$$\tau = \frac{L}{R_{eq}} \quad R_{eq} = \left[\frac{1}{R_2} + \frac{1}{R_3} \right]^{-1}$$

$$\tau = 13,1ms$$

$$I_e(\infty) = \frac{E}{R_3}$$

$$I_e(\infty) = 4,5$$

$$I(\infty) = I^A + I_e$$

$$I^A(\infty) = -7A$$

$$I^E(\infty) = -4,5A$$

$$I(\infty) = -11,5A$$

$$I(\tau) = K e^{-t/\tau} + I(\infty)$$

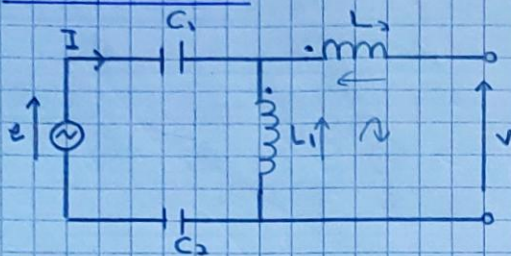
$$-7 + 11,5 = K$$

$$K = 4,5$$

$$I(2\tau) = -10,9A$$

26/06/2003

ESENCIZIO 1



$$e(t) = 14.1 \sin(314t + \pi/4)$$

$$L_1 = 120 \text{ mH}$$

$$L_2 = 100 \text{ mH}$$

$$C_1 = 100 \text{ } \mu\text{F}$$

$$C_2 = 150 \text{ } \mu\text{F}$$

$$M = 100 \text{ mH}$$

$$Z_{1C} = -31,85j \Omega$$

$$Z_{eq} = Z_{1C} + Z_{2C} + Z_{1L}$$

$$Z_{2C} = -21,23j \Omega$$

$$Z_{eq} = -15,4j \Omega$$

$$Z_{1L} = 37,68j \Omega$$

$$Z_{2L} = 31,4j \Omega$$

$$0,74$$

$$I = \frac{E}{Z_{eq}} \quad I = 0,65 \text{ } \underline{135} \text{ A}$$

$$V = Z_{1L} \cdot I \quad V = 24,4 \text{ } \underline{135} \text{ V}$$

$$I_e = I + I_3$$

$$S = -6,48j \text{ VA}$$

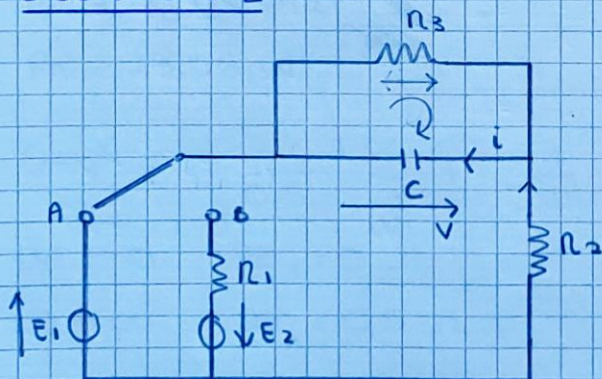
$$I_e - I_3 = I$$

$$V_2 = j\omega M I_1 + j\omega L_2 I_2$$

$$V_1 = j\omega L_1 I_1 + j\omega M I_2$$

$$V = V_1 - V_2 \quad V = 4 \text{ } \underline{135} \text{ V}$$

ESENCIZIO 2



$$E_1 = 12 \text{ V}$$

$$E_2 = 6 \text{ V}$$

$$R_1 = 10 \Omega$$

$$R_2 = 7,5 \Omega$$

$$R_3 = 15 \Omega$$

$$C = 1000 \text{ } \mu\text{F}$$

$$I_3 = -0,53$$

$$V(0^+) = V(0^-)$$

$$V(0^-) = -R_3 I$$

$$I = \frac{E_1}{R_3 + R_2} = 0,533 \text{ A}$$

$$V(0^-) = -8 \text{ V}$$

$$I_e(0^+) = I(0^+) + I_3(0^+)$$

$$V(0^+) = -8 \text{ V}$$

$$I_3(0^+) = \frac{V(0^+)}{R_3} = -0,53 \text{ A}$$

$$I_e = \frac{E}{R_{eq}}$$

$$I = 1,3 \text{ A}$$

$$\tau = C \cdot R_{eq} \quad \tau = 8,1 \cdot 10^{-3} \text{ s}$$

$$V(\infty) = -8 \text{ V}$$

$$\tau_2 = C \cdot R_{eq} \quad \tau_2 = 5 \text{ ms}$$

$$V^* = V_2 \quad V^* = 11,143 \text{ V}$$

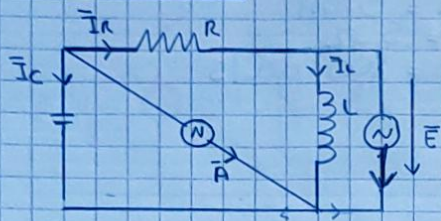
$$I_2 = I_e = \frac{E_2}{R_{eq}} = 0,7429 \text{ A}$$

$$V(\tau) = K e^{-t/\tau} + V(\infty)$$

$$V(\tau^-) = 1,2 \text{ V}$$

29/06/2009

ESERCIZIO 1



$$\bar{A} = 12 \angle 90^\circ \text{ A}$$

$$E = 9 \angle -30^\circ \text{ V}$$

$$R = 0,5 \Omega$$

$$C = 4 \text{ mF}$$

$$L = 4 \text{ mH}$$

$$Z_C = -0,5j \Omega$$

$$Z_L = 2j \Omega$$

$$\omega = 500 \text{ rad/s}$$

$$I_C^E = - \frac{E}{Z_C}$$

$$I_C^E = 4,5 \angle 60^\circ \text{ A}$$

$$I_E = 10,06 \angle -3,43^\circ \text{ A}$$

$$I_E + I_L = -I_C$$

$$I_C = -I_E - I_L$$

$$I_C^E = 12,73 \angle -164,99^\circ \text{ A}$$

$$I_R^E = 12,73 \angle 15^\circ \text{ A}$$

$$I_L^A = 0 \text{ A}$$

$$I_C^A = 8,48 \angle 135^\circ \text{ A}$$

$$I_R^A = 8,48 \angle 45^\circ \text{ A}$$

$$I_C^{\text{TOT}} = 11,22 \angle -124^\circ \text{ A}$$

$$I_L^{\text{TOT}} = 4,5 \angle 60^\circ \text{ A}$$

$$I_R^{\text{TOT}} = 6,85 \angle -23,2^\circ \text{ A}$$

$$Q_C = -63 \text{ Var}$$

$$Q_L = 40,5 \text{ Var}$$

$$P_R = 23,46 \text{ W}$$

$$Q = -22,5 \text{ Var}$$

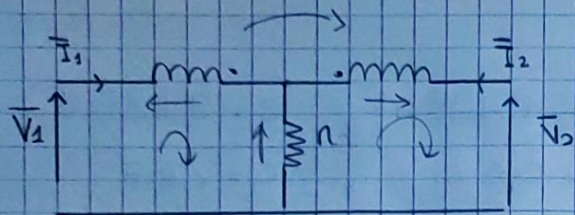
$$P = 23,46 \text{ W}$$

$$Z_{eq}^E = \left[\frac{1}{0,5 - \frac{j}{\omega C}} + \frac{1}{j\omega L} \right]^{-1}$$

$$\omega \rightarrow 0 \quad Z_{eq} = 0 \Omega$$

$$\omega \rightarrow \infty \quad Z_{eq} = 0,5 \Omega$$

ESERCIZIO 2.



$$\begin{aligned} L_1 &= 4 \text{ mH} \\ L_2 &= 5 \text{ mH} \\ R &= 5 \, \Omega \\ \omega &= 800 \text{ rad/s} \end{aligned}$$

$$j\omega L_1 = 3,2j \, \Omega$$

$$j\omega L_2 = 4j \, \Omega$$

$$Z_{11} = 5 + 3,2j$$

$$Z_{22} = 5 + 4j$$

$$Z_{12} = Z_{21} = \left(\frac{V_1}{I_2} \right)_{I_1=0}$$

$$V_1 = R_2 \cdot I_2$$

$$Z_{12} = Z_{21} = R_2 = 5$$

$$Z = \begin{bmatrix} 5 + 3,2j & 5 \\ 5 & 5 + 4j \end{bmatrix}$$

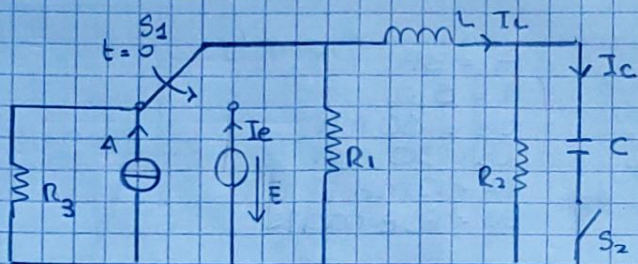
$$M = 4 \text{ mH}$$

$$V_1 = j\omega L_1 I_1 + j\omega M I_2 + R I_2$$

$$Z_{12} = Z_{21} = 5 + 3,2j \, \Omega$$

$$Z = \begin{bmatrix} 5 + 3,2j & 5 + 3,2j \\ 5 + 3,2j & 5 + 4j \end{bmatrix}$$

ESERCIZIO 3



$$\begin{aligned} E &= 9V \\ A &= 6A \\ R_1 &= 4\Omega \\ R_2 &= 4\Omega \\ R_3 &= 4\Omega \\ L &= 3H \\ C &= 4F \end{aligned}$$

$$I_1(0^-) = I_1(0^+) = I_2(0^-)$$

$$I_2(0^-) = A \cdot \frac{R_{eq}^*}{R_{eq}^* + R_2} = \cancel{6} \cdot \frac{2}{4 + \cancel{2}} = 2A$$

$$I_1(0^-) = I_1(0^+) = 2A$$

$$I_e(0^-) = 0A$$

$$V_{R_2}(0^-) = R_2 \cdot I_2(0^-)$$

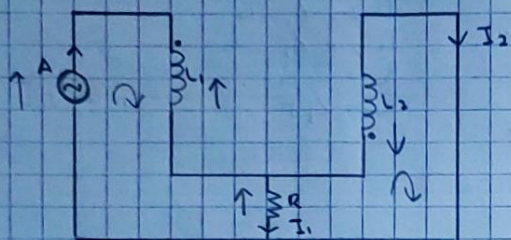
$$V_{R_2}(0^-) = 8V$$

$$I_2(0^+) = I_1(0^+) = 2A$$

$$V_{R_2}(0^+) = 8V$$

20/06/2005

ESENCIZIO 1



$$A = 3A$$

$$R = 5 \Omega$$

$$L_1 = 10 \text{ mH}$$

$$L_2 = 20 \text{ mH}$$

$$M = 12 \text{ mH}$$

$$\omega = 628 \text{ rad/s}$$

$$j\omega L_1 = 6,28j \Omega$$

$$j\omega L_2 = 12,56j \Omega$$

$$j\omega M = 7,536j \Omega$$

$$M = 0$$

$$V_A = j\omega L_1 \cdot A + R \cdot I_1$$

$$I_1 = A \cdot \frac{j\omega L_2}{j\omega L_2 + R}$$

$$I_1 = 2,787 \angle 21,7^\circ A$$

$$V_A = 27,3 \angle 61,64^\circ V$$

$$I_2 = A - I_1 \quad I_2 = 1,1 \angle -68,3^\circ A$$

$$S = V \cdot I^*$$

$$P_A = 38,9 \text{ W}$$

$$Q_A = 42,02 \text{ Var}$$

$$S_A = 81,9 \angle 61,64^\circ \text{ Va}$$

$$M = 12 \text{ mH}$$

$$V_A = j\omega L_1 I_A + R \cdot I_1$$

$$I_A = I_1 + I_2$$

$$j\omega L_1 = j\omega L_1 I_A + j\omega M I_2$$

$$j\omega L_2 = j\omega M I_A + j\omega L_2 I_2$$

$$R \cdot I_1 = j\omega M I_A + j\omega L_2 I_2$$

$$R I_1 = j\omega M I_A + j\omega L_2 (I_A - I_1)$$

$$R I_1 + j\omega L_2 I_1 = (j\omega M + j\omega L_2) I_A$$

$$I_1 = I_A \frac{j\omega M + j\omega L_2}{R + j\omega L_2}$$

$$I_1 = 4,41 \angle 21,7^\circ A$$

$$I_2 = 1,97 \angle -124,1^\circ A$$

$$V_A = 37,73 \angle 21,6^\circ V$$